ARITHMETIC-ALGEBRA
PART II

ARITHMETIC-ALGEBRA *PART—II*

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ARITHMETIC-ALGEBRA

PART II

Mathematics for Middle Schools

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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Foreword

There is today a ferment in science education. The rapid developments in science and technology in recent years have made it necessary to think of changes in curriculum, textbooks and methods of teaching mathematics and science in our schools. While this is generally true of all countries in the world, it is more so for India, if we wish to bridge the distance which separates us from the advanced countries. Unless our pupils in the schools learn more mathematics and better mathematics, it will not be possible for them to help the country in developing technology at a faster rate. But to do this in the same time span of ten or eleven years in the school, it is necessary to think of new ways of teaching mathematics. If a scientific attitude has to develop, its base must be built on conceptual understanding and reasoning rather than on retention and reproduction of bits of information.

Science in some form or the other, either as general science or as everyday science, is being taught in all our schools at the middle stage. It has, however, been felt by all concerned that this is far from adequate, as it does not provide a firm foundation for the understanding of basic concepts of physics, chemistry and biology. The Education Commission (1964 66) has observed that "the general science approach to the teaching of science which has been very widely adopted at the elementary stage during the last 10 years has not proved successful as it tends to make science appear somewhat formless and without structure and runs counter to its methodology A disciplined approach to science learning would, it is felt, be more effective in providing the necessary scientific base to the young pupils."

The UNESCO Planning Mission which visited this country in 1964 also made similar recommendations for effecting improvements in the teaching of science and mathematics in Indian schools. Following these recommendations, the Department of Science Education in the National Council of Educational Research and Training has undertaken a major project to upgrade science and mathematics teaching starting from the middle school stage. A new curriculum in science and mathematics for the middle stage as separate disciplines of physics, chemistry, biology and mathematics has been prepared and textbooks, teachers' guides and curriculum guides are being developed. It is expected that this curriculum would equip those who leave school at the close of the middle stage with enough basic knowledge of the sciences and mathematics and their applications. The middle stage covers three years of school education. All the materials developed under this project are being tried out in selected schools and on the basis of the experience gained, the instructional materials are being refined and further developed for application throughout the country. The first part of the series of text materials, teachers' guides and curriculum guides has already been published and is being used in some selected schools of Delhi and in the Central Schools all over the country. The present series of text materials is for the second year course of the middle stage. The curriculum has a practical bias and the knowledge of science is given through experimental activities to be performed by teachers and pupils. Every care has been taken to present a correct and up-to-date view of science and to see that the examples used are drawn from Indian situations. A variety of indigenous equipment and apparatus have been designed and produced by the Central Science Workshop of the National Council to make better instructional materials available to teachers.

The syllabi and text materials in mathematics are based on the structural approach, laying emphasis on mathematical concepts and generalizations. Arithmetic as such has not been given much prominence, but is included as an integrated part of wider generalization—algebra. The isolated topical treatment has been linked and integrated as applications of algebraic principles. The courses have been reorganized so as to reduce the routine computational work and make it more functional. In geometry the approach has been practical and logical, starting with undefined terms, defined terms, definitions and axioms and then going on to the theorems. Use has been made of symmetry and reflection to simplify proofs. An attempt has been made to correlate the various branches. Thus it is possible to teach more mathematics in the limited time available. The main objective is to develop intuition, thinking and logical reasoning while learning mathematics.

The National Council is grateful to UNESCO experts who assisted the Department of Science Education on this project. It is also thankful to the Director of Education, Delhi Administration, who very kindly provided facilities to try out the curricular materials in selected schools in Delhi. The National Council will welcome comments and suggestions from teachers and others interested in science education for the improvement of these materials.

SHIB K. MITRA
Joint Director
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New Delhi
20th January, 1968

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CHAPTER 1

Rational Numbers

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§1. A Rational Number

You have already learnt about fractions, decimals and mixed numbers in primary classes and also in Class VI. You have also learnt the operations of addition, subtraction, multiplication and division on these numbers. All these numbers can be put under one general heading **Rational Numbers**. Here, in this class we will give a more systematic treatment of these numbers, just as given for whole numbers and integers in Class VI.

While learning about natural numbers you learnt that subtraction and division were not always possible. The set of integers made the operation of subtraction always possible, and to make the division possible, rational numbers were introduced. So a fraction is an indicated quotient of two natural numbers. Similarly, **a rational number** *is also an indicated quotient of two integers. Hence a rational number is of the form* $\frac{p}{q}$ *where* p *and* q *are integers and* $q \neq 0$*. A rational number consists of integers (including zero), positive and negative fractions (including decimals).*

1. Point out which of the following numbers are rational numbers :

   $\frac{3}{4}$, $-2{\cdot}5$, $\frac{5}{2}$, $\frac{4}{1}$, $\frac{2}{0}$, $\frac{3}{x}$ $(x \neq 0)$ and $\frac{a}{b-b}$

   (b is an integer).

2. Mark the following numbers on the number line .

(*a*) $\frac{1}{8}, \frac{-1}{5}, 1\frac{1}{2}, \frac{3}{4}, \frac{-5}{16}$.

(*b*) 1·3, ·8, -1·4, -0·7, 1·9.

3. If a, b, c are three rational numbers and if $a > b$ and $b > c$ then $a > c$. Explain it by taking at least two examples, one from common fractions and other from the decimals.

4. How will you know from the number line that a rational number is greater or less than another rational number ?

5. How many rational numbers will there be between : (*i*) 2 and 3 (*ii*) -1 and -2 (*iii*) 0 and 1. Do you see any end to these numbers ?

Explain your answer with the help of the number line also.

6. How many decimals will there be between : (*i*) 0 and -1 (*ii*) 5 and 6 (*iii*) -3 and -4. Draw a number line and explain your answer.

7. Between two rational numbers there is no end to rational numbers. For example, between $\frac{1}{3}$ and $\frac{1}{4}$ we can put as many rational numbers as we like such as

$$\frac{\frac{1}{3}+\frac{1}{4}}{2}=\frac{7}{24}.$$

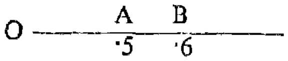
Other numbers are

$$\frac{7}{22}, \frac{7}{23}, \frac{7}{25}, \frac{7}{26}, \frac{7}{27}, \frac{14}{43}, \frac{14}{45}, \ldots\ldots\ldots\ldots\ldots$$

Can you write a few rational numbers between $\frac{3}{4}$ and $\frac{1}{2}$?

8. Write rational numbers between ·5 and ·6.
(For example, ·51, 511, ·501, 52...........)

It will be more clear from the number line. The point A



represents ·5 and point B represents ·6. Between A and B there may be countless points representing countless rational numbers.

9. Can you always find a point on the number line for each rational number ?

10. Can you always find a rational number for each point on the number line ?

11. (*a*) Find the integral value of x for which $\frac{3}{x}$ is a natural number.

(*b*) Find an integral value of x for which $\frac{5}{x}$ is an integer.

(*c*) Find those integral values of x for which $\frac{6}{x}$ is

(*i*) natural number (*ii*) an integer.

12. (*a*) Find the least integral value of x for which $\frac{x}{3}$ is a natural number. How many values of x will there be if x is also a natural number ?

(*b*) Find the least integral values of x for which $\frac{x}{3}$ is an integer.

(*c*) Find any two values (one positive and the other negative) of x for which $\frac{x}{5}$ is an integer. Will there be an end to such values of x for which $\frac{x}{5}$ is an integer ?

13. Suggest values of a and b such that $\frac{a}{b}$ may be a whole number ; a and b being natural numbers.

14. Can you suggest any rule by which you can always find 'if $\frac{a}{b}$ is a whole number, a and b being natural numbers.'

§2. Comparison of Rational Numbers

(*a*) *Through a Number Line*

It is easy to compare numbers with the help of a number line because the number line preserves the *order* of the rational numbers. On the number line these numbers are arranged from left to right in ascending order.

(*b*) *Through Arithmetical Processes*

There are two cases (*i*) rational numbers having the same denominator (*ii*) rational numbers having different denominators.

If the denominators of two positive rational numbers (fractions) are the same, fraction with the greater numerator is greater than the other. For negative rational numbers the case will be just opposite, *i.e.*, if two negative fractions have the same denominator ; the term with greater numerator will be less than the other.

Example 1

$$\frac{5}{8} > \frac{3}{8} \text{ but } \frac{-5}{8} < \frac{-3}{8}.$$

If the denominators of two fractions are not the same, they should be changed to fractions having the same denominator. For this we should know **Equivalent Fractions.** We already know that $\frac{a}{b}$, $\frac{ac}{bc}$ are equivalent fractions. Now if two fractions $\frac{3}{4}$ and $\frac{5}{6}$ are to be compared, change the fractions to the same denominator 12 (l.c.m. of 4 and 6).

Example 2

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \text{ and } \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\text{Since} \quad \frac{10}{12} > \frac{9}{12} \quad \text{so} \quad \frac{5}{6} > \frac{3}{4}$$

Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equal

i.e., $\frac{a}{b} = \frac{c}{d}$ if $a.d = b\,c\ (b \neq 0, d \neq 0)$.

For example, $\frac{2}{3} = \frac{10}{15}$ since $2 \times 15 = 3 \times 10$.

This property enables us to simplify the rational numbers.

Rule I. **If a, b, c, d are any whole numbers and b and d $\neq$ 0, then $\frac{a}{b} > \frac{c}{d}$ if and only if a.d $>$ b c.**

Rule II. **If a, b, c, d are any whole numbers and b and d $\neq$ 0, then $\frac{a}{b} = \frac{c}{d}$ if and only if a.d $=$ b.c.**

§3. Absolute Value of a Rational Number

Suppose there are two numbers 5 and -5. For each of these numbers we shall refer the number 5 as its **absolute value.** The absolute value is also called **numerical value.** In such cases the number is written between two vertical bars,

i.e., $|5| = |-5| = 5$

Similarly we can write

$$|x| = |-x| = x \text{ when } x > 0$$

and $$|x| = |-x| = -x \text{ when } x < 0$$

where x is any rational number

15. Compare the following numbers .

$\frac{3}{5}$ and $\frac{2}{3}$; $\frac{7}{8}$ and $\frac{11}{16}$; $\frac{5}{6}$ and $\frac{7}{8}$

16. Replace the sign ? by one of the appropriate signs >, =, <

(*i*) $\frac{9}{10}$? $\frac{7}{8}$ (*ii*) $\frac{7}{16}$? $\frac{5}{11}$

(*iii*) $\frac{5}{13}$? $\frac{3}{7}$ (*iv*) $\frac{7}{13}$? $\frac{21}{39}$.

17. Find if the following statements are true or false.

(*i*) $\frac{-5}{8} < \frac{-7}{8}$ (*ii*) $\frac{4}{5} > \frac{8}{10}$

(*iii*) $\frac{7}{12} > \frac{13}{24}$ (*iv*) $\frac{3}{5} = \frac{9}{15}$.

18. Solve the following equations :

(*i*) $|x| = 2$ (*ii*) $|x| = 3$

(*iii*) $\left|\frac{5}{6}\right| = x$ (*iv*) $|x + 1| = 4$

(*v*) $\left|x - \frac{1}{2}\right| = \frac{2}{3}$ (*vi*) $|x - \cdot 3| = 1{\cdot}2$.

19. (*i*) $|2 - x| = 1$ (*ii*) $x = \left|1\frac{1}{2}\right|$

(*iii*) $x = |3{\cdot}4|$ (*iv*) $|3x| = 6$

(*v*) $4x = |5|$ (*vi*) $\frac{x}{2} = |\cdot 8|$

20. If x is a whole number find those values of x for which the following inequalities are true :

(*i*) $x < 2$ (*ii*) $2x < 3$ (*iii*) $(3x - 1) < 2$.

§4. Operations on Rational Numbers

(*a*) *Addition*

You have already done problems on addition of rational numbers in Class VI.

Example 3

Suppose we have to add $\frac{a}{b}$ and $\frac{c}{d}$ $(b \neq 0, d \neq 0)$

$$\frac{a}{b} + \frac{c}{d} = \frac{a\,d}{b\,d} + \frac{b.c}{b\,d} = \frac{a\,d + b.c}{b.d}$$

where b and $d \neq 0$.

21. Is $\frac{5}{3} + \frac{7}{8} = \frac{7}{8} + \frac{5}{3}$?

22. Show that $2{\cdot}3 + 7{\cdot}9 = 7{\cdot}9 + 2{\cdot}3$.

23. Examine if $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ $(b, d \neq 0)$.

24. Add the following numbers in two ways :

(*i*) $\frac{3}{4} + \frac{5}{4} + \frac{2}{3}$ (*ii*) $\frac{7}{3} + \frac{5}{13} + \frac{2}{3}$

(*iiii*) $7{\cdot}387 + 4{\cdot}987 + 2{\cdot}613$ (*iv*) $\cdot 456 + 872 + \cdot 128$.

25 Add :

(*i*) $\frac{2}{3} + \frac{7}{5}$ (*ii*) $\left(\frac{-5}{6}\right) + \left(\frac{-7}{12}\right)$

(*iii*) $\cdot 356 + \cdot 780$ (*iv*) $\frac{8}{15} + \left(\frac{-3}{4}\right)$

(*v*) $\left(\frac{-1}{4}\right) + \frac{1}{2}$.

You will find that the sum in each case is always a rational number.

26. Add :

(*i*) $\frac{2}{5} + 0$ (*ii*) $-3{\cdot}5 + 0$

(*iii*) $0 + \frac{a}{b}$ $(b \neq 0)$ (*iv*) $0 + 0$.

So we have the following properties of addition in the case of rational numbers :

I. Addition is commutative,

i.e., $$\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$$

if a, b, c, d are integers and b and $d \neq 0$.

II. Addition is associative,

i.e., if a, b, c, d, e, f are any integers and $b, d, f \neq 0$, then

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$$

III. Addition is closed over the set of rational numbers (closure property), i.e., if a, b, c, d are any integers and $b, d \neq 0$, then $\frac{ac}{bd}$ is always a rational number.

IV. Zero is an identity element for the set of rational numbers, i.e., if we add zero to any rational number the sum is the same rational number.

V. $\frac{2}{5}$ and $\frac{-2}{5}$; 8·3 and $-8{\cdot}3$; $\frac{a}{b}$ and $\left(-\frac{a}{b}\right)$ form pairs of opposites and sum of these opposites is zero as in the case of integers.

Hence, $$\left(\frac{a}{b}\right)+\left(-\frac{a}{b}\right)=\left(\frac{a}{b}\right)-\left(\frac{a}{b}\right)=0.$$

(*b*) *Subtraction*

Let us examine the properties of addition for subtraction.

27. Is $\frac{2}{3}-\frac{1}{4}$ the same as $\frac{1}{4}-\frac{2}{3}$?

28. Is 4·5 — 2·3 the same as 2·3 — 4·5 ?

From the above we see that subtraction is not commutative.

Similarly, you can verify that it is not associative.

The closure property is true in this case also because the difference of two rational numbers is always a rational number.

29. Solve :

(*i*) $\frac{7}{23} - 0$ (*ii*) $11{\cdot}2 - 0$

(*iii*) $\frac{a}{b} - 0\ (b \neq 0)$ (*iv*) $0 - 0$

30. Subtract :

(*i*) $2{\cdot}3 - 1{\cdot}8$ (*ii*) $7{\cdot}812 - 4{\cdot}907$

(*iii*) $7001{\cdot}201 - 588{\cdot}875$ (*iv*) $1{\cdot}61 - 5{\cdot}43$

(*v*) $0{\cdot}01 - 0{\cdot}10$ (*vi*) $2{\cdot}9 - 2{\cdot}9$

31. (*i*) $5\frac{1}{2} - 3\frac{3}{4}$ (*ii*) $2\frac{1}{2} - 5\frac{1}{4}$ (*iii*) $4 - 3\frac{8}{9}$.

32. (*i*) $\frac{a}{b} - \frac{c}{d}\ (b, d \neq 0)$ (*ii*) $\frac{p}{q} - \frac{r}{s}\ (q, s \neq 0)$.

33. Illustrate through examples that subtraction is not associative.

34. Write down the opposites of :

(*i*) $\frac{-17}{6}$ (*ii*) $5\frac{3}{7}$ (*iii*) $12{\cdot}7$

(*iv*) $-6{\cdot}9$ (*v*) $\frac{e}{f}$ (*vi*) $-\frac{x}{y}$

(*c*) *Multiplication*

Multiplication of two rational numbers is defined in the same way as in the case of fractions, *i.e.*, if $a, b, c; d$ are any integers such that $b \neq 0, d \neq 0$.

Then $$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}.$$

It means that for multiplying two rational numbers you should multiply their numerators and denominators separately. The result will be a rational number of the form p/q $(q \neq 0)$, such that p is the product of the numerators and q is the product of the denominators. In Class VI you have already done questions on the multiplication of rational numbers. Let us examine here the properties of multiplication.

(*i*) *Commutative Property.*

Take any two rational numbers, *e.g.*, $\frac{2}{3}$ and $\frac{4}{5}$.

Now $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Also $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

Hence, $\frac{2}{3} \times \frac{4}{5} = \frac{4}{5} \times \frac{2}{3}$.

The fact can be verified by taking other examples also. Hence, **multiplication is commutative,** *i.e.*, **if a, b, c, and d are any integers (b, d $\neq$ 0), then**

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}.$$

(*ii*) *Associative Property.*

Multiply : $\frac{2}{3} \times \frac{4}{5} \times \frac{7}{11}$

$$\frac{2}{3} \times \frac{4}{5} \times \frac{7}{11} = \frac{8}{15} \times \frac{7}{11}$$

$$= \frac{8 \times 7}{15 \times 11} = \frac{56}{165}.$$

Now multiply, $\frac{2}{3} \times \frac{4}{5} \times \frac{7}{11}$ by first multiplying the last two terms. The result will be

$$\frac{2}{3} \times \left(\frac{4}{5} \times \frac{7}{11}\right) = \frac{2}{3} \times \frac{28}{55}$$

$$= \frac{2 \times 28}{3 \times 55} = \frac{56}{165}.$$

The two results are the same

$$\left(\frac{2}{3} \times \frac{4}{5}\right) \times \frac{7}{11} = \frac{2}{3} \times \left(\frac{4}{5} \times \frac{7}{11}\right).$$

Verify the fact by taking other examples also. Hence **multiplication is associative,** *i.e.*, **if a, b, c, d, e, f, are any integers (b, d, f $\neq$ 0), then**

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right).$$

(*iii*) *Distributive Property.*

Simplify · $\frac{4}{5} \times \left(\frac{2}{3} \times \frac{3}{4}\right)$.

It can be done in two ways. One way is first to add $\frac{2}{3} + \frac{3}{4}$ and then to multiply the sum by $\frac{4}{5}$. In this we get the result as

$$\frac{4}{5} \times \frac{17}{12} = \frac{4 \times 17}{5 \times 12}$$

$$= \frac{68}{60} = \frac{17 \times 4}{15 \times 4} = \frac{17}{15}.$$

The other way is first to multiply $\frac{4}{5}$, $\frac{2}{3}$ and also $\frac{4}{5}$, $\frac{3}{4}$ and then to add the two products. In this way the result will be

$$\frac{4}{5} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} = \frac{8}{15} + \frac{4 \times 3}{5 \times 4}$$

$$= \frac{8}{15} + \frac{3 \times 4}{5 \times 4}$$

$$= \frac{8}{15} + \frac{3}{5} = \frac{17}{15}.$$

The two results are the same.

$$\therefore \qquad \frac{4}{5} \times \left(\frac{2}{3} + \frac{3}{4}\right) = \frac{4}{5} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4}.$$

Verify the same by taking some more examples. Hence the **multiplication is distributive over addition, i.e., if a, b, c, d, e and f are any integers (b, d, f $\neq$ 0), then**

$$\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}.$$

(*iv*) *Closure Property.*

Multiply : $\frac{7}{5} \times \left(\frac{-2}{3}\right)$. The result will be

$$-\frac{(7 \times 2)}{(5 \times 3)} = \frac{-14}{15}$$

which is again a rational number. Similarly, you can take other examples and see for yourself that the product of two rational numbers is always a rational number.

Hence multiplication is closed over the set of rational numbers, *i.e.*, **if a, b, c, d are any integers (b, d $\neq$ 0), then**

$$\frac{a}{b} \times \frac{c}{d} = \frac{a.c}{b.d}$$

and $\frac{ac}{bd}$ is always a rational number.

(*v*) *Identity Element.*

Multiply : (*i*) $\left(\frac{7}{9}\right) \times 1$ (*ii*) $\left(\frac{-7}{9}\right) \times 1$.

(*i*) $\left(\frac{7}{9}\right) \times 1 = \left(\frac{7}{9}\right) \times \left(\frac{1}{1}\right) = \frac{7 \times 1}{9 \times 1} = \frac{7}{9}$

(*ii*) $\left(\frac{-7}{9}\right) \times 1 = \left(\frac{-7}{9}\right) \times \left(\frac{1}{1}\right)$

$$= \frac{-7 \times 1}{9 \times 1} = \frac{-7}{9}.$$

In each case the product is always equal to the multiplicand. Take other examples and verify the fact. Therefore if one of the two given rational numbers is unity (*i,e*, 1) the product of those two numbers will always be equal to the other number. Hence, **if a, b are integers (b ≠ 0), then**

$$\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}.$$

∴ **Identity element for multiplication is 1**

(*vi*) *Reciprocal.*

Multiply the pairs of numbers :

$$\frac{5}{6} \text{ and } \frac{6}{5},\ \frac{17}{13} \text{ and } \frac{13}{17},\ \left(\frac{-53}{64}\right) \text{ and } \left(\frac{-64}{53}\right).$$

In each of above cases the result will be 1.

When the product of two rational numbers is 1, each one is called **the reciprocal of the other. Hence if** a, **b, c, d are any integers and if** $\frac{a}{b} \times \frac{c}{d} = 1$, **then** $\frac{c}{d}$ **is the reciprocal of** $\frac{a}{b}$ **and** $\frac{a}{b}$ **is the reciprocal of** $\frac{c}{d}$, *i.e.*, $\frac{c}{d} = 1 \div \frac{a}{b}$ **and** $\frac{a}{b} = 1 \div \frac{c}{d}$ (a, **b, c, d ≠ 0). Also** $1 \div \frac{a}{b}$ **or** $\frac{b}{a}$ **is the reciprocal of** $\frac{a}{b}$ **where** $\frac{a}{b}$ **is any rational number (a, b ≠ 0).**

(*vii*) *Multiplication by zero.*

You know that

$$\frac{7}{9} \times 0 = 0\ ;\ \left(\frac{-5}{6}\right) \times 0 = 0.$$

Therefore, just like integers, multiplication by zero also gives the product as zero in the case of rational numbers. Hence, **if a, b are any integers (b ≠ 0) then**

$$\frac{a}{b} \times 0 = 0 \times \frac{a}{b} = 0.$$

35. Find the value of x in the following equations (x is any non-zero rational) :

(*i*) $\left(1\frac{3}{8}\right) \times \left(5\frac{3}{17}\right) = 5\frac{3}{13} \times x$

(*ii*) $(x) \times (-6{\cdot}9) = (-6{\cdot}9) \times (2{\cdot}4)$

(*iii*) $\left(\dfrac{23}{29}\right) \times (x) = \left(\dfrac{-37}{-19}\right) \times \left(\dfrac{23}{29}\right)$

(*iv*) $\left(\dfrac{15}{18}\right) \times \left(\dfrac{32}{27}\right) = (x) \times \left(\dfrac{15}{18}\right)$.

36. Perform the following operations in two ways :

(*i*) $2\frac{1}{2} \times 1\frac{7}{8} \times 1\frac{3}{5}$ (*ii*) $1{\cdot}5 \times {\cdot}4 \times (-3{\cdot}7)$

(*iii*) $\frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6}$ (*iv*) $2\,8 \times 1{\cdot}2 \times 5$.

37. Simplify by using distributive law :

(*i*) $7\frac{1}{2} \times \frac{3}{4} + 7\frac{1}{2} \times \frac{1}{4}$

(*ii*) $\frac{8}{9} \times \frac{14}{5} - \frac{8}{9} \times \frac{4}{5}$

(*iii*) $6{\cdot}5 \times 3{\cdot}7 + 6{\cdot}5 \times 6{\cdot}3$

(*iv*) $8{\cdot}9 \times 4{\cdot}7 - 8{\cdot}9 \times 3{\cdot}7$.

38. Write down the reciprocals of (c, d, p and x are integers $\neq 0$)

$$\left(\frac{-2}{3}\right), \left(\frac{7}{5}\right), (4), \left(\frac{c}{d}\right), (x), \left(\frac{3}{x}\right), (px), (7{\cdot}9), \frac{0}{x},$$

39. Find the following products : (p, q are integers $\neq 0$)

$$\left(\frac{53}{49}\right) \times \left(\frac{-49}{53}\right); \left(\frac{p}{q}\right) \times (1);$$

$$\left(\frac{-2}{3}\right) \times \left(\frac{-3}{2}\right); \left(\frac{p}{q}\right) \times 0.$$

40. Solve the following equations (x is always a rational number) :

(*i*) $\frac{x}{3} = \frac{2}{9}$ (*ii*) $\frac{x}{2{\cdot}5} = 4$

(*iii*) $\frac{x}{-5} = {\cdot}2$ (*iv*) $\frac{x}{a} = \frac{1}{a}$ (a is an integer).

(*d*) *Division*

Division has the same meaning for rational numbers as it has for fractions and natural numbers. Since division and multiplication are inverse operations, the methods of performing division can be derived for the extended set of rational numbers also. For doing the operation of division in the case of fractions we multiplied the dividend by the reciprocal of the divisor. Similar is the method for rational numbers.

If a, b, c, d are any non-zero integers, then

$$\frac{a}{b} \div \frac{c}{d} \text{ means } \frac{a}{b} \times \frac{d}{c}.$$

Let us repeat the rules we learnt for division of integers in Class VI as applied to non-zero rational numbers :

(*i*) The quotient of two positive rational numbers is a positive rational number, *e.g.*,

$$\left(\frac{+7}{8}\right) \div \left(\frac{+5}{4}\right) = +\left(\frac{7}{8} \times \frac{4}{5}\right).$$

(*ii*) The quotient of a positive rational number and a negative rational number is a negative rational number, *e.g.*,

$$\left(\frac{+7}{8}\right) \div \left(\frac{-5}{4}\right) = -\left(\frac{7}{8} \times \frac{4}{5}\right).$$

(*iii*) The quotient of a negative rational number and a positive rational number is a negative rational number, *e.g.*,

$$\left(\frac{-7}{8}\right) \div \left(\frac{+5}{4}\right) = -\left(\frac{7}{8} \times \frac{4}{5}\right).$$

(*iv*) The quotient of two negative rational numbers is a positive rational number, *e.g.*,

$$\left(\frac{-7}{8}\right) \div \left(\frac{-5}{4}\right) = +\left(\frac{7}{8} \times \frac{4}{5}\right).$$

41. Do you always get a rational number when you divide a rational number by another non-zero rational number ? Take a few examples and study the result. Can you say that division is closed over the set of rationals excluding zero ?

42. Is division commutative for rational numbers ? Prove through examples that it is not.

43. Is division associative for the set of rational numbers ? Prove through examples that it is not.

44. Does the division distribute addition ? Prove through examples that it does not.

45. If you divide any rational number by 1 ; the quotient will be always equal to the dividend. Prove it through examples.

46. Divide :

(*i*) 0 by $\frac{3}{5}$ (*ii*) 0 by (-0·5) (*iii*) 0 by $\frac{17}{13}$, and

(*iv*) 0 by $\frac{a}{b}$ where a and b are integers $\neq 0$.

Remember division by zero is not defined.

§5. Proof of the Commutative Property of Addition

Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two rational numbers in which a, b, c, d are any integers and b, d are not equal to zero. We have to prove that

$$\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}.$$

By the definition of addition of rational numbers ;

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

$$\text{and} \quad \frac{c}{d} + \frac{a}{b} = \frac{cb + ad}{db}.$$

Now as ad and cb are integers by the closure property of multiplication of integers, we have

$ad + cb = cb + ad$ (by the commutative property of addition of integers)

and $bd = db$ (by the commutative property of multiplication of integers)

$\therefore \quad \frac{ad + cb}{bd} = \frac{cb + ad}{db}$ (by the definition of equality of rational numbers)

Hence $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.

The proofs of other properties are similar and hence left for student to prove himself.

47. Solve the following problems :

(*a*) A note-book contains 12 sheets of paper. How many sheets of paper will m note-books contain ?

(*b*) A father is 30 years older than his son. How old is the father, if his son is p year old ?

(*c*) A ship goes m km in 4 hours. Calculate the average speed of the ship.

(*d*) The width of a rectangle is equal to x cm and its length is half as much again as its width. Calculate the perimeter and the area of the rectangle.

As a result of solving each of the above problems you obtained expressions containing digits, letters, signs of operations and signs of equality or inequality. What are such expressions called ?

48. (*a*) a, b, c and d are four numbers and m is their mean. Write the formula for finding m.

(*b*) Compute the formula for the calculation of the area of each shaded figure (Figs. 1.1 and 1.2) given below. Use necessary notations.

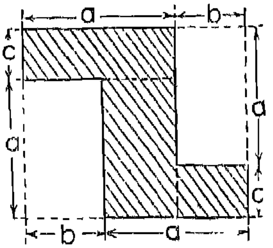


Fig. 1.1.

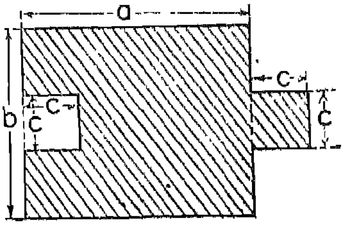


Fig. 1.2.

(*c*) From a village a cyclist starts for a town s km distant from the village with a speed of a km/h. At the same time another cyclist starts from the town to the village with a speed of b km/h. After riding for 5 hours the cyclists were at a distance of d km from each other. Express d in terms of s, a and b.

(*d*) The speed of a boat in still water is equal to a km/h and the speed of the current of water is equal to b km/h ($a > b$). Calculate the speed of the boat in favour of the current and also against it. What will it be if $a = b$ and $a < b$?

49. Given $a : b = c$. From the formula calculate a in terms of b and c. Can b have any value, whatsoever ?

50. From the formula $C = \dfrac{5(F - 32)}{9}$, where C denotes the number of degrees in Celecius scale, F denotes, the number of degrees in Fahrenheit scale, express F in C and find it, when $C = 0°, 20°\ ; -15°, -40°$.

51. Express as a formula :

A number a when divided by a number b gives 4 as the quotient and 2 as the remainder.

Instead of the word *formula* for these expressions we can also use the word *algebraic expression.*

An expression, which contains numbers, denoted by either digits or letters and connected by signs of operations, is called an **algebraic expression.**

52. Find the numerical value of the following algebraic expression :

$$N = \frac{2x - y}{(x - y)(x + y)} \text{ if } x = 3{\cdot}3 \text{ and } y = 0{\cdot}7.$$

53. Complete the following table if $y = \dfrac{3x - 2}{x - 1}$:

$x =$	0	$1\frac{1}{2}$	2	3	$^{-}1$	$^{-}2$	
$y =$							

54. Find a formula for calculating the values in the following problems :

(*a*) Find p per cent of a number a.

(*b*) Find the unknown number, if p per cent of it is equal to another number b.

55. After 20 per cent reduction in the price a book was available for Rs. m. Find its cost price before, reduction.

56. Number a is greater than the other number b by 5. By what per cent is the number a greater than number b ?

57. Two workers can do a piece of work in m hours. One of them spends t hours on this work. How much time will be needed to get the remaining work done by the other worker ?

58. A reduction of 10 per cent in the price of mangoes enables me to purchase 5 more for Rs. 20. Find the price per mango before and after the reduction.

59. The average age of a family of 5 members, 3 years ago, was 17. A baby having born the average age at present is the same. Find the age of the baby.

60. The average weight of 12 boys is 40 kg. But it is increased by 0 5 kg when a new boy joins them. Find the weight of the new boy.

61. The average rainfall in a city on Tuesday, Wednesday, Thursday and Friday was 18·9 mm and on Wednesday, Thursday, Friday and Saturday it was 17 mm. If the rainfall on Saturday was 20 mm, find the rainfall on Tuesday.

62. In a factory there are 16 workers. A new worker joins on a salary of Rs. 100 p.m. By the addition of this new worker, the average salary of the workers becomes less by Re. 1 than the average salary of the 16 workers. Find the average salary of the 16 workers.

63. A school has worked for 200 days this year. Madhava was absent for 15 days. Find his percentage of attendance this year.

64. The price of rice falls from 80P per kg to 75P per kg. What is the reduction per cent ?

65. The population of India in 1961 was 439,233,000 and the number of literates was 105,333,000. Find the rate of literate persons per thousand.

66. A horse was sold for Rs. 525. If the gain on the horse was 5 per cent, find its cost price.

67. The price of daily articles of consumption have gone up by 10 per cent. By what percentage a man should reduce his consumption of the articles so that his budget does not increase ?

68 The heights of two vertical posts are 5 m and 6 m and their shadows measured at the same time are 7·5 m and 9 m respectively. Do these measurements taken in the given order, form a proportion ?

69. A tower 25 m high casts a shadow 20 m long. At the same time a boy casts a shadow 1·2 m long. How tall is he ?

70. 25 electric lamps, each 40 watt, consume Rs. 8 worth of electricity in a month. What will be the expenditure of electricity consumed in a month, if 20 lamps, 60 watt each, be used ?

71. A garden is to be constructed round a village tank. The villagers can level the ground in 7½ days, if they work 6 hours a day. They want to do the work in 5 days. How many hours a day should they work ?

72. Last week rice was sold at Re. 1 per kg. This price has since been increased by 10 per cent. Find the current price.

73. A shopkeeper buys 6000 litres of paint at Rs. 1·25 a litre and spends Rs 250 for advertisement and Rs 650 for other items of expenditure. He sells the entire lot at Rs. 1·61 a litre. Find the profit per cent that he makes.

74 Govind buys 600 apples at Rs. 20 a hundred and spends Rs. 5 for conveyance and Rs 3 for preserving them. He retails them and makes a profit of 25 per cent on the outlay. He sells 400 at 25 paise each. At what price does he sell the rest ?

75. A village panchayat levies a tax on the villagers for providing lights in the streets. A villager's yearly income is Rs. 2825 and he pays a tax of Rs 226. Find the rate of tax per rupee that he pays, if the tax is in the ratio of the annual income

76. Suresh borrows Rs. 2500 at 8 per cent per year and clears the debt after 3 years. How much does he pay to clear the debt ?

77. A sum of money doubles itself in 10 years. In how many years will it be fourfold at the same rate of simple interest ?

78. A man has to pay Rs. 1100 at the end of 5 years. He agrees to pay his debt through equal annual instalments. Find the amount of each instalment, if the rate of simple interest is 5 per cent.

79. Find the time in which the simple interest at Rs. 625 at 5 per cent per annum will be Rs. 93·75.

80. What sum of money will amount to Rs. 336 in 3 years at 4 per cent per annum simple interest ?

81. If Rs. 1200 amounts to Rs. 1308 in 3 years ; find the rate of interest.

82. Two horses are sold for the same price. On one there is a gain of 10 per cent and on the other there is a loss of 10 per cent. Find the percentage of loss or gain on both.

83. A man buys a car for Rs. 12000 and pays the money in cash. Another man buys the same car for Rs. 12495 to be paid after one year. If the rate of simple interest is 5 per cent, find who is the gainer.

84. Use the formula $I = \frac{P \times n \times r}{100}$; (P = Principal, n number of years, r = rate per cent per annum and I simple interest) to find time when the simple interest on Rs. 800 at the rate of $2\frac{1}{2}$ per cent is Rs. 70 only.

85. A man borrows Rs. 660 from each of the three persons. To one he paid Rs 148 extra, to the second Rs. 139 extra and to the third Rs. 109 extra as interest after 2 years. What would have been the rate of interest if he had borrowed the total money from one man and paid the same amount of interest as he paid to three persons ?

86. A trader marks his goods 40 per cent above cost price but allows a discount of 20 per cent How much per cent does he gain ?

CHAPTER II

Algebraic Expressions

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§1. Constants or Mathematical Objects

You have learnt about natural numbers, whole numbers, integers and rational numbers. They represent different sets of numbers. The set of rational numbers includes natural numbers, whole numbers, integers and positive and negative fractions also. Each number or member of this set always expresses one idea which is fixed and which cannot be changed, *e.g.*, the number 4 represents fourness, $\frac{3}{5}$ represents the idea of three fifthness and so on. To express these ideas in mathematical language certain numerals are used. These numerals are the denotations of these ideas. Mathematically we can say that they denote certain number ideas which are constant or a mathematical object can be denoted by any rational number,

$$e.g., \qquad 1, 4, \frac{7}{13}, 32, \frac{2}{7}.$$

There are other denotations also for these number ideas. You might have come across the denotation 'π'. It also denotes a constant. You will learn about the other constant numbers in the next class.

§2. Variable

Look at the formula for simple interest.

$$I = \frac{P \times r \times t}{100}.$$

If $P = 500$, $r = 4$, $t = 2$ we get

$$I = \frac{500 \times 4 \times 2}{100} = 40.$$

Give your own values to P, r and t and get the values of I accordingly. Here you see, you can give any arbitrary but fixed values to P, r and t and can get values of I. Such elements are called **variables**. Hence by a *variable we mean a letter denotation for an arbitrary but fixed value in the set attached for the variable.*

For example x, y, z, u, v......are variables, where each of these letters may be attached to any set, say set of natural numbers or integers or rational numbers.

Sometimes the variable may be attached to such smaller set as {2, 3, 4, 5, 6, 7} also. In this case the arbitrary value given to the letter will be one of these numbers. It will be discussed in article 4.

1. Point out the variables and the constants in the following :

$$\frac{3}{4},\ 4{\cdot}5x,\ \cdot 5y,\ z,\ \frac{7}{3}.$$

2. In ax, a has a value which does not change. Will you call it a constant or a variable ?
3. What can be the values of the variable x if $x < 5$ and x is always a whole number.
4. Write down the values of the variable y which is even and is always between 0 and 10.
5. A variable is prime and less than 50 but greater than 30 Write down its values.

§3. Exponents

Formulas Containing Exponents

NOTION OF POWER : While solving some problems, sometimes we may get the product of several equal factors. Here are two problems to be examined.

6. Suppose the side of a square a cm and its area is S cm^2. Express S in terms of a. Find S, when $a = 1{\cdot}2$.

7. The edge of a cube is a cm and its volume is V cm^3. Express V in terms of a. Find V when $a = 4$.

The product of a number of equal factors is generally written in a shorter way as follows :

$3 \times 3 \times 3 \times 3 = 3^4$ (three raised to the power of 4),

$8 \times 8 \times 8 = 8^3$ (eight raised to the power of 3),

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$ (product of five factors, each equal to 2),

$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3)$ (product of 4 factors, each equal to -3),

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ (product of six factors, each equal to 3).

In the last example the number 3 is called the *base* and the number 6 is called the *exponent*. The exponent indicates the number of times the base is taken as a factor in the product. A number equal to 3^6 is called a *power* of 3. In this case it is the sixth power of three.

Example 1

(*a*) $2^5 = 32$, 2 is the base and 5 is the exponent ; 2^5 or 32 is the fifth power of the number 2

(*b*) The formulas for the area of square or the volume of a cube, deduced in problems 6 and 7 can be written as :

$S = a^2$ (a raised to the power two, or 'a raised to the second power').

$V = a^3$ ('a raised to the power three' or 'a raised to the third power').

From these examples, it is clear why the second power of a number is (sometimes) called as the square of the number and the third power as the cube of the number. The words 'squared' and 'cubed' originated with the use of these operations to obtain the areas of squares and volumes of cubes respectively. Instead of 'a raised to the second power' we say 'a squared'; instead of 'a raised to the power three', we say 'a cubed' or 'cube of a'. The exponent is omitted for the first power, *i.e.*

$$2^1 = 2,\ (-1{\cdot}3)^1 = -1{\cdot}3.$$

8. Calculate .

$$61^3,\ (-3)^5,\ (1{\cdot}44)^4,\ 2^7,\ (1{\cdot}95)^3,\ (3{\cdot}2)^5.$$

9. Find the values of the expressions $5a$ and a^5, when $a = 3{\cdot}2$, -3.

10. Draw up a table of squares of the natural numbers from 1 to 20.

11. Draw up a table of cubes of the natural numbers from 1 to 10.

12. Calculate 10^n, when $n = 1, 2, 3, 4, 5, 6$. What changes does the number, which is the power of 10, undergo with the increase of the exponent ?

13. Calculate $(0{\cdot}1)^n$, when $n = 1, 2, 3, 4, 5, 6$. What changes does the number, which is the power of $0{\cdot}1$, undergo with the increase of the exponent ?

14. Compare the expressions :

(*a*) 10^2 and 2^{10} (*b*) 2^4 and 4^2 (*c*) 3^4 and 4^3.

15. a and b are natural numbers. Is it in general true that $a^b = b^a$? Give some values to a and b and see the result.

16. Calculate and draw the possible conclusions :

(*a*) $1^4, 1^7, 1^{12}, 1^n$ (n is a natural number)

(*b*) $0^5, 0^0, 0^{11}, 0^n$ (n is a natural number)

Power of a Negative Number

Suppose we are asked to calculate $(-3)^5$ Try to find at first sight the sign of the result.

Calculate :

$$(-2)^2, (-5)^5, (-7)^2, (-1)^5.$$

We see that raising a negative number to a power with an even exponent gives us a positive number, while raising a negative number to a power with an odd exponent gives us a negative number.

17. Find which of the following numbers are positive, and which are negative :

$(-0{\cdot}2)^9$, $(3{\cdot}1)^5$, $(-7)^2$, 3^7, $(3{\cdot}16)^2$, $(-1)^1$, $(2{\cdot}07)^3\,(-1)^{11}$ and $(0)^8$.

18. Which is greater ?

(*a*) $(-3{\cdot}1)^5$ or 0 (*b*) 0 or $(-2{\cdot}4)^1$

(*c*) 7^1 or $(-5)^7$ (*d*) 1^1 or $(-2)^1$

(*e*) 1^5 or $(-2)^5$.

19. If $a^7 = 0$, can a be a negative number ?

20. If $a^6 = 0$, which of the relations are true ?

$$a < 0, a > 0, a = 0.$$

21. Can the inequality $x^2 > x$ be true when :
(*a*) $x > 0$, (*b*) $x < 0$ and (*c*) $x = 0$?

22. Is it always true that $x^2 = (-x)^2$?

23. Mark points on a number line corresponding to all the values of x in whole numbers satisfying the inequality $1 \quad 2^2 < 110$.

§4. Order of Operations in the Expressions containing Power

We use the following order of operations : First simplify the quantities raised to a power ; then simplify the quantities involving multiplication and division in the order in which they are written and then simplify the quantities involving addition and subtraction, as they occur. If you have brackets, first do all the operations in brackets and then remove the brackets.

Example 2

Calculate the value of the expression $(-8)^3 - 2^2 \times 3$.

(1) $(-8)^3 = -512$
(3) $4 \times 3 = 12$
(2) $2^2 = 4$
(4) $-512 - 12 = -524$.

In a shorter way it can be written as :

$$(-8)^3 - 2^2 \times 3 = (-512) - 4 \times 3$$
$$= -512 - 12 = -524.$$

24. Calculate :

(a) $-8^3 + (-2)^2 \times 3$
(c) $(-2 - 3)^3$
(b) $(-2)^3 + (-3)^3$
(d) $-(2 + 3)^3$.

25. What will be the order of operation to calculate the value of each of the following expressions (suppose the values of the letters are given) :

(a) $(2x)^3 - 1$
(b) $2x^3 - 1$
(c) $(2x - 1)^3$
(d) $a^2 - b^3$
(e) $(a - b)^2$
(f) $a^3 - 2b^4$.

26. Write down the numbers, given in the following sentences, in the form of the product of a number and the power of ten, according to the following pattern :

$$5000 = 5 \times 1000 = 5 \times 10^3$$
$$12000000 = 12 \times 1000000 = 12 \times 10^6.$$

(a) the distance between the Earth and the Sun is 150000000 km,

(b) the population of India is approximately 430000000,

(c) out of the Earth's surface totalling 510000000 km^2, land occupies 149000000 km^2 and water occupies 361000000 km^2,

(d) a computer does 7200000 operations per hour,

(e) 1 cm^3 of gas contains 27000000000000000000 molecules.

27. Calculate and read out :

$$36 \times 10^5 , 4\ 5 \times 10^8, 14 > 10^9$$

28. $y = 2ab^3$ Calculate y, when $a = -1{\cdot}2$; $b = -1{\cdot}3$.

29 For approximate calculation of the volume of a haystack (Fig. 2.1), we use the formula :

$$V = \frac{nl^2}{72}$$

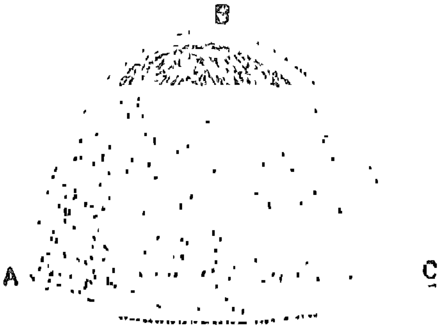


Fig. 2.1

where V is the volume of the haystack (in cubic metres) l the length of the circumference of its base (in metres), n the length of ABC (in metres). Calculate the volume of haystack when ·

(a) $l = 12$ m, $n = 18$ m (b) $l = 132$ dm, $n = 154$ dm.

30. A stone falling on the ground, covers the distance of s m in t seconds where s is given by $s = 4{\cdot}9t^2$. Calculate s, if $t = 3, 8, 5, 20$.

31. The length of a rectangular parallelopiped is a cm ; the width is equal to the length and its altitude is b cm and the area of the total surface of the parallelopiped is S cm². Express S in terms of a and b.

Find S when $a = 12$, $b = 15$.

32. Find the value of the expression a^n, when :

(*a*) $a = -0{\cdot}036$, $n = 2$ (*b*) $a = -84{\cdot}5$, $n = 1$

(*c*) $a = -1$, $n = 8$ (*d*) $a = 0$, $n = 9$.

33 The speed of light is 3×10^5 km/sec Calculate and read the result.

34. Compare the expressions :

(*a*) 3^3 and 3^5 (*b*) $(-3)^3$ and $(-3)^5$

(*c*) $0{\cdot}2^2$ and $0{\cdot}2^5$ (*d*) $(-0{\cdot}2)^3$ and $(-0{\cdot}2)^5$

(*e*) $(-3)^3$ and $(-3)^2$ (*f*) 3^5 and 5^3.

35. (*a*) Which is greater, $2a$ or a^2, if $a > 0$?

(*b*) If $b > 0$, which of the inequalities is true :

$b^5 < 0$, $b^5 > 0$ or $b^5 = 0$?

36. Find the number, square of which is 81. How many such numbers can you find ?

37. $10 < x^2 < 20$ and x is a whole number. Find x.

38. Mark on a number-line all the whole numbers satisfying the inequality $x^2 < 30$.

39. Find the value of the expression :

$$\frac{(m^2 + n^2) + 2mn}{5} \quad \text{when } m = -0{\cdot}6,\ n = -0{\cdot}4.$$

40. If $y = \dfrac{(x^2 + 6x + 9) - (x^2 - 6x + 9)}{12}$;

fill in the table :

x	-2	-1·2	-1	0	1·2
y					

41. $s = \dfrac{(a^2 + 2a) - (a^2 - 2a)}{a}$. What is the value inappropriate for a ? Fill in the table :

a	-0·5	-3	1	100	1000
s					

42. The edge of a cube is equal to a cm and its total surface is S cm^2. Express S in terms of a. Find S when $a = 1{\cdot}3$

43. The base of a rectangular parallelopiped is a square with the side equal to a m, the altitude of the parallelopiped is h m and its volume is Vm^3. Express V in terms of a and h. Find h using the formula obtained.

§5. Algebraic Expression

We learnt in Class VI that an **algebraic expression** *contains terms connected by the operations of addition and subtraction ; and a term contains variables and constants with the operations of multiplication and division. A single term is also called an* **expression**.

The examples of expressions are

$$3 + 4,\ 2\ 3 - 1{\cdot}4 + \frac{3}{4},\ 3x + 1,\ \frac{x}{y},\ x.y \text{ etc.}$$

Arithmetic value of the expression depends upon the value of the variable or the values of the variables used For example, the value of the expression $3x + 1$ will be 1 if $x = 0$. Now if x has any value from the set {2, 3, 4, 5, 6, 7} the values of the expression will be 7, 10, 13, 16, 19 and 22. In this case the expression cannot have any other values.

[**Note.** In this book, unless otherwise stated, the variable will always have values from the set of rational numbers]

44. Point out the number of terms in the following expression :

$$3{\cdot}5x + 1\frac{1}{2} - \frac{3x^2y}{z} \times ab.$$

45. Write an expression having 3 terms and involving at least two variables.

46. Write at least 4 single term expressions involving the operations of multiplication and division.

§ 6. Polynomials

Consider the algebraic expression $2a^2 + 3a$ where a is any rational number. By using distributive law we can write it as $a(2a + 3)$

So $$2a^2 + 3a = a(2a + 3) \qquad \text{...(A)}$$

We can also write it as

$$2a^2 + 3a = (a^2 + 4) \times \frac{2a^2 + 3a}{a^2 + 4} \qquad \text{...(B)}$$

The simple expression $2a^2 + 3a$ can also be written in the form $(a^2 + 4) \times \frac{2a^2 + 3a}{a^2 + 4}$.

Let us examine the two forms of the expression. The expression $2a^2 + 3a$ involves integers 2 and 3, the variable a and the indicated operations of addition and multiplication but the expression $\frac{2a^2 + 3a}{a^2 + 4}$ involves integers 2 and 3, the variable a and the indicated operations of addition, multiplication and division. The first expression does not involve division. This type of expression is called a *Polynomial.*

Hence :

A **polynomial** *in one variable is a sum of terms each of which is a product of a number called 'coefficient' and powers of one variable where the exponents are whole numbers, e.g.,*

$$-2x^3 + \frac{2}{3}x + \frac{7}{8},\ \cdot 5x + 1 - 3\cdot 2x^2.$$

If the variables are more than one, it is a polynomial in more than one variable, e.g.,

$$a + x,\ 3x^2y + xy^2 + \frac{2}{3}xy + 4y^2.$$

(*i*) *If the coefficients are integers it is a polynomial over integers, e.g.,*

$$4x^3 - 3x + 2.$$

(*ii*) *If the coefficients are rational numbers it is a polynomial over rational numbers, e.g.,*

$$\frac{3}{4}x - \frac{2}{7}x^2 + 1{\cdot}5x^3 - 1.$$

Note. In this book we will not go beyond polynomials over rational numbers and variables will also have values in rational numbers only.

§ 7. Degree of a Polynomial

(*i*) In the case of polynomials in one variable the largest exponent of the variable denotes the degree of the polynomial. For example, in

$$2x - 3 + 4x^5 + 7x^2$$

the degree of the polynomial is 5.

(*ii*) In the case of polynomials in more than one variable the sum of the exponents of the variables denotes the degree of the term and the degree of the term with the largest degree will be the degree of the polynomial. For example, in

$$2x^4y^3 - 3x^5y^6 + 4x^8 - 7x^3y^5 - 5y^{10}$$

there are 5 terms. The degree of the first term is $4 + 3 = 7$, of the second is $5 + 6 = 11$, of the third is $8 + 0 = 8$, of the fourth is $3 + 5 = 8$, and of the fifth is $0 + 10 = 10$. The second term has the largest degree, *i.e.*, 11. Hence the degree of the polynomial is 11.

§ 8. Properties of Polynomials

You can verify by taking examples the following properties of the polynomials over rationals :

(*a*) *Properties of Addition*

(1) Sum of two polynomials is always a polynomial.

(2) If A and B are two polynomials then $A + B = B + A$.

(3) If A, B and C are three polynomials, then

$$(A + B) + C = A + (B + C).$$

(4) There exists a zero polynomial such that

$$A + 0 = 0 + A = A$$

where A is a polynomial.

(5) To every polynomial, say, A, there exists an opposite polynomial $(-A)$ such that

$$A + (-A) = 0.$$

(*b*) *Properties of Multiplication*

(1) Product of two polynomials is always a polynomial.

(2) If A and B are two polynomials, then

$$A.B = B.A$$

(3) If A, B and C are three polynomials, then

$$(A.B)\,C = A\,(B.C)$$

(4) There exists a unit polynomial such that

$$A \times 1 = A,$$

where A is a polynomial.

(5) If A, B and C are three polynomials then

$$A \times (B + C) = A \times B + A \times C.$$

Let us find the reciprocal of a polynomial. Let $x^2 + 1$ be a polynomial. Then $(x^2 + 1) \times \frac{1}{x^2 + 1}$ will be equal to 1. So the reciprocal of $x^2 + 1$ is $\frac{1}{x^2 + 1}$. But $\frac{1}{x^2 + 1}$ is not a polynomial. Generally if A is a polynomial, then

$$A \times \frac{1}{A} = \frac{1}{A} \times A = 1 \; (A \neq 0).$$

So A and $\frac{1}{A}$ are reciprocals of each other. But $\frac{1}{A}$ is not a polynomial. **Hence a polynomial has no reciprocal.**

§ 9. Rational Expressions

Let us take division of a polynomial by another polynomial.

Suppose you have to divide $x^2 + 3x + 1$ by $x^2 - 6x + 1$. The quotient will be $\frac{x^2 + 3x + 1}{x^2 - 6x + 1}$. It is not a polynomial. In general if A and B are two polynomials, $A \div B$ or $\frac{A}{B}$ **is not always a polynomial. So division is not always possible in the case of polynomials.**

Perhaps you remember that division was not always possible in the case of integers, so we extended our system of numbers to rational numbers Similarly, to make the division possible, we extend here from polynomials to *Rational Expressions.*

Hence an expression which involves rational numbers and variables **with at most the operations** *of addition, subtraction, multiplication and division is called* **a rational expression over rational numbers.**

For example, $\frac{3}{t}$, $\frac{3x + 1}{5}$, $2a - 3b$, $\frac{x^2 + 1}{t + 1}$ are all rational expressions provided denominator is not zero in any case.

In this chapter you will read about polynomials and their operations and in Chapter V you will read about the operations on rational expressions including division.

47. Put down several rational algebraic expressions making use of the :

(*a*) operations of addition and subtraction
(*b*) operations of multiplication
(*c*) operations of multiplication and division
(*d*) operations of division and using exponents
(*e*) operations of addition, subtraction and multiplication
(*f*) all operations.

We will call a rational algebraic expression a *whole expression*, if it does not include the operation of division by literal numbers. For example, the algebraic expressions a, $a + x$, ax^2 and

$\frac{ax}{5} - \frac{3x^2}{8}$ are whole algebraic expressions : and algebraic expressions like $\frac{2}{3x}$, $\frac{2 - x^2}{a}$, $\frac{4}{a}$, $\frac{3x}{a - 1}$ are *fractional expressions* having the operation of division with literal numbers.

48. Put down several whole and fractional algebraic expressions.

49. Separate the whole and the fractional expressions :

$$2a - 3, \frac{-2b}{3}, \frac{3x^2 - 4x + 5}{3}, \frac{a + c}{4b}, \frac{-a}{2x - 1},$$

$$-4 + \frac{3b}{5}, 3 - \frac{3}{5b}, 2x - 3x + 5, - \frac{(a^3 - b)^2 + 3}{7a},$$

$$x^9 + y^2 \text{ and } \frac{a^2 - ab}{15}.$$

Let us consider the whole algebraic expressions, containing the operations of multiplication and raising to power only.

For example :

$$9a, \frac{2}{3} b, 0{\cdot}5a^2c, -0{\cdot}7x^2y^2, 3, d, ad^3, - a \text{ and } 0{\cdot}5.$$

Such algebraic expressions having only one term are called *Monomials.*

An algebraic expression, consisting of only one letter, is also considered as a monomial. Numbers alone are also monomials.

Therefore, the expressions 3, d, $- a$, $-0{\cdot}5$ are monomials.

50. Put down several monomials in your exercise book.

51. What are the numerical coefficients of the following monomials ?

$$7y, 0{\cdot}5x, \frac{2}{3} b, m, - b, \frac{a}{3}, \frac{3y}{5}, \frac{-x}{2}$$

While writing monomials, numerical coefficients are generally written first and then come the literal numbers. The letters are generally arranged in alphabetical order.

52. Put down the following monomials properly :

$3ma^2x$, $a^2(-5)x^2$, $-b^2ac$, $-5bc^3$, $2ma$, $3m^2ax^3$, $4(-x^2)$, $5ac(\frac{1}{2}a^2b)$.

Point out the numerical coefficient of each monomial.

53. When $a = 2$ and $x = -1$, calculate the values of the following monomials :

(*a*) $-a^2x$ (*c*) $-a(-4)a^2x^3x^2$
(*b*) $-a \times 5a^3x^4$ (*d*) $(ax)^2 \times 3(-1)a$.

54. Which of the following expressions are monomials ?

(*a*) $10a^2bc^2$ (*b*) $\frac{a^2}{10}$
(*c*) $(a+b)^2$ (*d*) $\frac{a+b}{5}$
(*e*) $\frac{7ac^3}{11}$ (*f*) $0{\cdot}6 + a^2$.

55. Having considered x as the main letter of the following monomials, point out the coefficient of x or any power of x in each case :

(*a*) $1{\cdot}2x^2$ (*b*) $-0{\cdot}65x^3$ (*c*) $6ax$ (*d*) $11b^2x$
(*e*) x (*f*) $-x$ (*g*) $-0{\cdot}3ax^2$.

56. Arrange the following monomials in the decreasing order of powers of a :

$$-8abx,\ 0{\cdot}2xy,\ 15a^2x^2,\ -5a^3.$$

57. Write down the sum of the following monomials :

(*a*) $2a$ and $5x$ (*b*) $0{\cdot}5b$ and $-y$ (*c*) $-3ac$ and $-5xy$.

58. Write down the difference of the following monomials :

(*a*) $8y$ and $9ab$ (*b*) $3b$ and $-0{\cdot}2$ (*c*) $-7ax$ and ay.

59. Add monomial $3x^2y$ to the difference of $3b$ and c.

60. Subtract monomial $5xy$ from the sum of $2x$ and $-7m$.

Several monomials, joined together by signs of plus and minus, form a new algebraic expression, called a *Polynomial*.

Thus an algebraic sum of several monomials is called a **polynomial.**

61. Put down several polynomials.

62. Which of the following expressions are polynomials ?

(1) $\frac{a^2 - 2a + 3}{16}$ (2) $\frac{0{\cdot}2 + \frac{b}{3}}{3a}$

(3) $0{\cdot}31y$ (4) $3a^2x$

(5) $a(x + y) \times \frac{2}{3}x$ (6) $\frac{b(x + a)^2}{c}$

63. Put down the following polynomials as algebraic sum of the monomials :

(*a*) $-6x^5 + 3x^4 - 12x^2 + 5$

(*b*) $2{\cdot}5x^4 - 18x^3y - 16x^2y^2 - 3xy^3$.

64. Find the numerical values of the following polynomials :

(*a*) $a^2 + 3ab + b^2$ if $a = -\frac{1}{2}$; $b = \frac{2}{3}$

(*b*) $2a^4 - ab + 2b^2$ if $a = -1$; $b = \frac{1}{2}$

65. Arrange the following polynomials so that the power of the letter mentioned in each goes on decreasing :

(*a*) $3ax^2 - 5a^3x + 7ax^4 - 8a^2x^3 + x^5$ according to the powers of x.

(*b*) $0{\cdot}2a^2x^2y - 0{\cdot}6a^3 - \frac{3}{4}ax^4 + \frac{5}{6}xy^2$ according to the powers of a.

66. Arrange the polynomial

$$\frac{5}{8}b^2xy^2 - \frac{5}{9}b^3x^2y^3 - 8y + 0{\cdot}1bx^3$$

so that the power of the letter b goes on increasing.

67. Put down several binomials and trinomials.

68. The side of a square is equal to q. Calculate the perimeter of the square.

69. The perimeter of an equilateral triangle is equal to p. Calculate each side of the triangle.

70. A car consumes a litres of petrol for each kilometre of the journey. The first day the car went 400 km, the second day 500 km and the third day 600 km. How much petrol was spent in the car within these three days? (Solve by *two* methods).

§ 10. Simplification of Similar Terms of Polynomials

The same monomials, or monomials which are different only in coefficients, are called *similar*.

For example, monomials $3a^2mc$ and $3mca^2$; $4a^3b$ and $6a^3b$, $2xy^2$ and $\frac{2}{3}xy^2$ are similar.

71. Put down several similar monomials.

72. Put down several monomials similar to monomial $-0{\cdot}7a^2x$.

73. Are the monomials bx^2y, $-bx^2y$, and $\frac{2}{3}bx^2y$ similar?

74. Among the monomials ax^2, $7abx^2$, $\frac{-2}{3}ab$, $0{\cdot}3abx^2$, $-ax^2$, ab, find similar monomials.

§ 11. Algebraic Sum of Similar Monomials

For example,

$$7m + 3m - 2m = (7 + 3 - 2)\,m = 8m.$$

Reducing an algebraic sum of similar monomials to one term identical with the sum is called **simplification of similar terms** *or* **reduction of similar terms.**

75. Reduce to one term (Orally):

(1) $-8m - 3m$ (2) $-2q + 2q$

76. Reduce to one term only (Orally):

(1) $-4m^3 + 10m^3 - 8m^3$ (2) $-25k^4 - 32k^4 + 48k^4$

Simplify :

77. (1) $\frac{2}{3}x^2 + \frac{5}{6}x^2 - \frac{1}{2}x^2$

(2) $1{\cdot}5n^6 - 0{\cdot}9n^6 + 2n^6$

78. $2y^2 - 3y + 2y - y^2$

79. $5ab - 4a^2b^2 - 8ab^2 + 3ab - ab^2 - 4a^2b^2 + 2a^2b^2 + ab^2$

80. (1) $0{\cdot}4a^2x - \frac{ax^2}{5} - \frac{1}{3}a^2x + 0{\cdot}3ax^2$

(2) $3(x - y) - 8(x - y) + 4(x - y)$

81. Solve the following equations :

(1) $-3 - 5x + 20 + 2x = 5$

(2) $\frac{2}{9}b - 5 - \frac{7}{18}b + 18 = 10.$

82. Obtain formula for the calculation of perimeters of the figures given below :

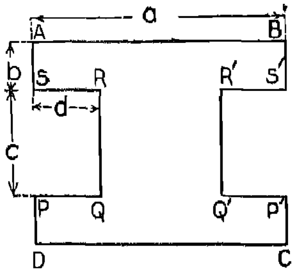


Fig. 2.2

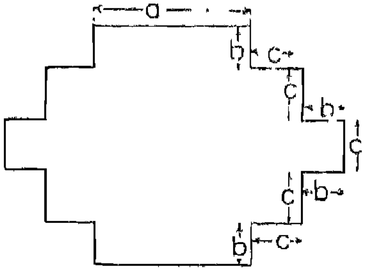


Fig. 2.3

§ 12. Addition of Monomials and Polynomials

(*a*) *Addition of Monomials*

Let us add the monomials :

$$5a^2, {}^{-}8xy, {}^{-}2a^2, {}^{+}10xy.$$

As a result we will obtain an algebraic sum of these monomials :

$$5a^2 + ({}^{-}8xy) + ({}^{-}2a^2) + ({}^{+}10xy),$$

Let us put down this sum in a different form as follows :

$$5a^2 - 2a^2 - 8xy^2 + 10xy.$$

Let us add the similar terms separately and obtain the final result as $3a^2 + 2xy$.

In order to add monomials, it is sufficient to put them one after the other with their signs (as an algebraic sum). Similar terms in the sum should be reduced to one term.

83. Simplify the following monomials (Orally) :

(1) $10a + (^{-}2a)$ (2) $(^{-}3d) + (^{-}2d)$

84. (1) $\left(\frac{-2}{3}\,c\right) + \left(\frac{-3}{4}\,c\right)$

(2) $(^{-}0{\cdot}3x^2) + (^{+}2x^2)$

85. Simplify :

(1) $\frac{3}{4}\,a^2b + \left(\frac{-2}{3}\,ax^2\right) + \left(\frac{-1}{2}\,a^2b\right) + \left(+1\frac{1}{3}\,ax^2\right)$

(2) $(^{-}1{\cdot}42ac) + (^{-}0{\cdot}96c) + (^{+}1{\cdot}54ac) + (^{-}0{\cdot}14c)$
$+ (^{-}0{\cdot}2ac) + (^{-}1{\cdot}6c)$

86. The sum of one-fourth and one-sixth parts of an unknown number is less than its half by 5. Find this number and check your result.

87. In order to prepare bronze, it is necessary to take 17 parts of copper, 2 parts of zinc and one part of tin. Find the quantity of each metal taken to prepare 2 kg of bronze.

88. The sides of a triangle are in the ratio of 5 : 4 : 3 and the perimeter of this triangle is 48 cm. Find the sides of the triangle.

89. When a pupil has read $\frac{2}{3}$ of a book, 240 pages are left to be read by him. How many pages are there in the book ?

90. Simplify the monomials :

(1) $5a^n + (^-2a^n) + (8a^{n+1}) + (^+6a^n) + (-a^{n+1})$

(2) $(^-9x^{\lambda+1}) + (^-4x^{\lambda}) + (^+12x^{\lambda+1}) + (5x^{\lambda}) + (x^{\lambda+1})$

91. Solve the following equations :

(1) $\left(\frac{^-3}{4}x\right) + (^-20) + \left(-1\frac{1}{2}x\right) - (^+8) = 2$

(2) $5x + (^-6{\cdot}2) + (^-6{\cdot}2x) + 2 = 1{\cdot}8$

92. Solve the following equations :

(1) $2x - 3x = 25$ (2) $3x - 2x = 10$

(*b*) *Addition of Polynomials*

93. Check the equality :

$$a + (b + c) = a + b + c$$

for all values of a, b and c. Formulate a rule for the addition of two numbers to any number.

94. Add a polynomial to a monomial in the following, using the rule of exercise 93 :

(1) $7a + (2x - 3{\cdot}5a)$ (2) $5 + \left(^-3y - 1\frac{2}{3}\right)$

95. Add the following polynomials, using the rule of exercise 93 :

(1) $11a^2 + 7a$ and $9a^2 - 5a$

(2) $1{\cdot}5a^2 + 2b^2$ and $2a^2 - 0{\cdot}3b^2$

In order to add polynomials, it is necessary to put down all terms of polynomials one after the other with their proper signs (as an algebraic sum).

If there are similar terms in the expression, it is necessary to reduce them to one term.

During the process of addition of polynomials we have to remove the brackets.

In order to remove brackets preceded by the sign of plus, it is necessary to put down all terms in brackets without bracket with their proper signs.

96. Reduce :

(1) $(10a - 6b + 5c - 4d) + (9a - 2b - 4c + 2d)$

(2) $(5x^2 - ax + a^2) + (3x^2 + 2ax - 3a^2)$
$+ (-4ax + 2a^2 - x^2)$

97. Simplify :

(1) $(1{\cdot}2a^4 + 3a^3 - 2a^2 - 0{\cdot}7a + 15) + (0{\cdot}8a^4 - 3a^3 + a^2$
$+ 0{\cdot}7a - 14)$.

(2) $(0{\cdot}9b^5 - 10b^3 + 1{\cdot}3b^2 - 2b - 3) + (0{\cdot}1b^5 + b^4 + 10b^3$
$+ 0{\cdot}7b^2 + 2b + 3)$

(3) $(8a^n - 2b^m + c) + (-4a^n - 5b^m - c)$

98. Solve the following equation :

$(2 + 5{\cdot}7p) + (18{\cdot}2 - 0{\cdot}855p) + 3{\cdot}45p = 36{\cdot}79$

Very often we have to use brackets for writing a polynomial or the part of a polynomial.

For example, $2x^2 + 76 - 8c$ may be written in the other way as $2x^2 + (76 - 8c)$.

In order to put polynomials in brackets preceded by a plus sign, it is necessary to put all the terms of the polynomial with their proper signs.

99. Put the following polynomials as a sum of two polynomials :

(1) $17a^4 - 8a^3x - 6a^2x^2 - ax^3$

(2) $3y^5 + 15y^4z - 12y^3z^2 + 2yz^4 - z^5$

100. Calculate by the simplest method the value of :

(1) $216{\cdot}25 + 853 + 783{\cdot}75 + 147$

(2) $419{\cdot}43 + 614{\cdot}65 - 219{\cdot}43 + 385{\cdot}35$

By arranging polynomials either in increasing or in decreasing order of powers, we can add them more conveniently through columns :

For example, let us add :

$(7x^4 - 8x^3 + 9x^2 - 10x + 11)$ and $(6x^4 + 2x^3 - 8x^2 - 3)$

Let us put down the terms in columns, arranged according to powers of x :

$$\begin{array}{rrrrr} 7x^4 - 8x^3 & + 9x^2 & - 10x & + 11 \\ 6x^4 + 2x^3 & - 8x^2 & & - 3 \\ \hline 13x^4 - 6x^3 & + x^2 & - 10x & + 8 \end{array}$$

101. Add the following polynomials by the column method :

(1) $(2a^4 - 5a^3 + 7a^2 - 9a + 12)$ and $(a^3 - 7a^2 + 3a - 5)$

(2) $(-11a^3 + 8a^2b - 7ab^2 + 6b^3)$, $(9a^3 - 7a^2b + 5ab^2 - 3b^3)$ and $(3a^3 - a^2b + 2ab^2 - 2b^3)$

(3) $(-1{\cdot}9a^3 - 1{\cdot}6a^2b + 2{\cdot}3ab^2 - 0{\cdot}3b^3)$, $(3{\cdot}2a^3 - 2{\cdot}4a^2b + 3{\cdot}1ab^2 - b^3)$ and $(-4a^2b + 1{\cdot}3b^3 - 5{\cdot}6ab^2 + 0{\cdot}7a^3)$

Let us consider two such polynomials, consisting of terms having the same modulus but opposite signs.

For example :

$$a^2 + 2ay - 5y^2 \text{ and } -a^2 - 2ay + 5y^2.$$

Two polynomials, consisting of terms, which have the same modulus and are of opposite signs, are called *opposite polynomials*.

102. Write down polynomials opposite to the following polynomials :

(1) $5a^8 - 7a^6 + 3a^4 - 2a^2 - 10$

(2) $-3b^5 + 4b^4 - 5b^3 + 6b^2 - 8$

103. Put down polynomials opposite to the sum of the polynomials :

(1) $(-4b^3 + 7b^2c + c^3)$ and $(2b^3 - 5c^3 - 8b^2c)$

(2) $(13x^4 + 9x^3 - 7x + 4)$ and $(7x^4 - 10x^3 + 5x^2 - 3x - 5)$

104. Put down polynomials opposite to the given polynomials and find the sum of the pairs of polynomials so obtained :

(1) $3a^4 - 5a^2b^2 + 7b$ (2) $5m^2 + 3m - 1$

What conclusion do you draw ?

105. One of the sides of a rectangle is $3m + 2n$, and the other side is greater than the first by $m - n$. Find the perimeter of the rectangle.

106. Reproduce the numbers 5372, 48936, 392745, as polynomials, arranging in decreasing powers of number 10.

For example :

$$5347 = 5 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 7$$

107. (1) Prove that the sum of two odd numbers is even.

(2) Prove that the sum of an odd number and an even number is odd.

108. (1) Prove that the sum of three consecutive odd numbers is divisible by 3.

(2) Prove that the sum of three consecutive even numbers is divisible by 6.

109. (1) Prove that the difference between a two-digit number and the number formed by reversing the digits is divisible by 9.

(2) Prove that the sum of a two-digit number and the number formed by the same digits in the reverse order is divisible by 11.

110. There are 4500 inhabitants in three settlements. The second settlement has inhabitants twice of those in the first settlement and the third settlement has 500 inhabitants less than those in the second settlement. How many inhabitants are there in each settlement ?

111. The side AB of a triangle ABC is greater than the side AC by 1 cm, and the side BC is greater than the side AB. 1 cm. by Perimeter of the triangle is equal to 15 cm. Find the length of each side.

§ 13. Subtraction of Monomials and Polynomials

(a) Subtraction of Monomials

Let us subtract $5m^2x$ from $9m^2x$. The result will be

$$9m^2x - (^{+}5m^2x).$$

Now $\quad 9m^2x - (^{+}5m^2x) = 9m^2x + (^{-}5m^2x)$

According to the rule of addition of monomials we will obtain :

$$9m^2x + (^{-}5m^2x) = 9m^2x - 5m^2x = 4m^2x.$$

In order to subtract a monomial, it is sufficient to add its opposite monomial to the minuend. Similar terms can be reduced to one term.

112. Subtract : (Orally).

(1) $^{-}7x$ from $4x$ (2) $^{-}3c^4$ from $-c^4$

(3) k from 0 (4) $^{-}7q$ from 0

113. Simplify :

(1) $0{\cdot}8a^2 - (^{+}1{\cdot}2a^2)$ (2) $^{-}9a^{2n+1} - (^{-}3a^{2n+1})$

(3) $^{-}0{\cdot}2m^2n - (^{-}1{\cdot}2m^2n)$

(4) $5a - (^{+}2a)$

114. Find the missing terms in the following equalities :

(1) $? + (^{-}8bc^3) = 2bc^3$

(2) $(^{-}15y^5) - ? = {}^{-}2y^5$

(3) $(^{-}18a^2b^7) - ? = 0$

115. (1) Add the difference of the monomials $4{\cdot}72n^4p$ and $^{-}1{\cdot}28n^4p$ to the sum of the monomials $2{\cdot}38n^4p$ and $^{-}1{\cdot}48n^4p$.

(2) Subtract the sum of the monomials $^{-}8{\cdot}39k^3n^4$ and $5{\cdot}29k^3n^4$ from the difference of the monomials $2{\cdot}57k^3n^4$ and $^{-}1{\cdot}43k^3n^4$.

116. Make the operations and simplify :

(1) $(^{-}1{\cdot}4a^3) - (^{-}0{\cdot}9a^3) + (^{-}1{\cdot}5a^3) + 2a^3$

(2) $\left(-\frac{p}{5}\right)-\left(-\frac{2p}{3}\right)-\left(+\frac{p}{4}\right)+\left(-\frac{p}{60}\right)$

(b) Subtraction of Polynomials

117. Check the equality :

$a - (b + c) = a - b - c$ when :
$a = 10,\ b = 4,\ c = 2.$

State the corresponding conclusion. Let us assume that we have to subtract the polynomial $a^2 + b^2 - 2ab$ from the polynomial $4a^2 - 2ab - b^2$.

Putting it in the correct form, we get :

$4a^2 - 2ab - b^2 - (a^2 + b^2 - 2ab)$.

Let us change the subtraction form to addition form by substituting the opposite polynomial for the subtrahend.

$4a^2 - 2ab - b^2 + (- a^2 - b^2 + 2ab)$.

According to the rule of addition we obtain :

$$4a^2 - 2ab - b^2 + (- a^2 + b^2 + 2ab)$$
$$= 4a^2 - 2ab - b^2 - a^2 - b^2 + 2ab = 3a^2 - 2b^2.$$

In order to subtract one polynomial from the other, it is enough to add all terms of the subtrahend with opposite signs to the minuend.

If there are similar terms in the expression, it is necessary to reduce them to one term.

118. Subtract :

(1) $2m - 3n$ from $5m + 6n$
(2) $6a^2 - 5a$ from $a^2 + 7a$.

119. Simplify :

(1) $(13x - 11y + 10z) - (-15x + 10y - 15z)$
(2) $(14ab - 37bc - 2cd) - (16bc + 11cd)$
(3) $1 - (-3p^2 + 2p + 1)$
(4) $0 - (- x^2 - y^2 - 1)$.

120. Subtract :

(1) $\left(a^2b^2 - \frac{1}{3}x^2y^2 + \frac{1}{12}ab - \frac{1}{4}\right)$

from $\left(\frac{1}{2}x^2y^2 - \frac{2}{3}ab - \frac{5}{6}a^2b^2 - 1\right)$

(2) $\left(3x^3 - \frac{2}{3} + \frac{1}{2}y^2 - \frac{1}{3}x^2y - 2y^2\right)$

from $\left(\frac{2}{3}x^3 - 3x^2y + \frac{1}{4}xy^2 - 2y^2 - 1\right)$

For subtraction also it is more convenient to arrange the polynomials in columns as was done in the case of addition

For example :

$$7a^3 + 10a^2 - 7a - 13 - (5a^3 - 2a^2 + 6a - 8)$$

is equivalent to :

$$\begin{array}{r}
7a^3 + 10a^2 - 7a - 13 \\
-\ 5a^3 + \ \ 2a^2 - 6a + 8 \\
\hline
2a^3 + 12a^2 - 13a - 5.
\end{array}$$

121. Solve by arranging the polynomials into columns :

(1) $(0{\cdot}8a^3b^2c - 0{\cdot}15a^4b^3c^2 + 1{\cdot}6a^5b^4c^3)$
$- (3{\cdot}2a^3b^2c + 2{\cdot}1a^4b^3c^2 - 0{\cdot}02a^5b^1c^3)$

(2) $(1{\cdot}4x^2 + 2{\cdot}24xy - 1{\cdot}5y^2)$
$-\left(\frac{-5}{8}xy - 10\frac{3}{4}\ \ x^2\ \ -1\frac{1}{2}y^2\right)$

122. Simplify :

(1) $(3a^{n+3} - 9a^{n+2} + 5a^{n+1} - 2a^n) - (a^n + 10a^{n+3} - 5a^{n+1}$
$- 7a^{n+2})$

(2) $(y^{n+1} - 5y^n - 7y^{n-1}) - (-6y^n - 8y^{n-1} + y^{n-2})$

123. $(7x^3 + 13x^2 - 19x - 25) - [(17 - 15x + 10x^2 - 4x^3$
$- (42 - 11x^3 - 2x^2 + 4x)]$

124. Solve the equations :

(1) $\left(2\frac{1}{2}x - 1\frac{1}{3}\right) - \left(3\frac{1}{4} - \frac{1}{3}x\right) + (-x - 1) = 8\frac{5}{8}$

(2) $\left(0\cdot15b - \frac{3}{4}\right) + (2\cdot36 - 0\cdot25) - \left(2 - \frac{1}{4}b\right) = 3\cdot12$

Very often it becomes necessary to remove brackets having the minus sign before it or to put polynomials into brackets having minus sign before them. In order to remove such brackets it is necessary to put down all the terms inside the brackets with opposite signs (without brackets).

In order to put a polynomial into brackets preceded by a minus sign, it is necessary to put down all the terms of the polynomial into the brackets with signs changed.

125. Put the extreme terms of the expression $2x + 5xy - 4xy^2 - y^3$ into brackets with the sign plus (+) before them and the middle terms into brackets with minus (−) sign before them.

126. Arrange the following polynomials so as to represent the difference of two polynomials :

(1) $13x^6 - 6x^4 - 5x^3 + 7x^2 + 8x + 4$
(2) $3y^2 - y + 1$.

127. Represent the trinomial $2a^2 - 6a + 4$ as the difference of two expressions with the subtrahend as $6a$.

128. Remove the brackets and simplify :

(1) $(4a^2 - ab - b^2) - (-a^2 + b^2 - 2ab) + (3a^2 - ab + b^2)$
(2) $(5a^2 - 3b^2) + [-(a^2 - 2ab + b^2) + (5a^2 - 2ab - 3b^2)]$
(3) $(3m + 5n) - [9m - \{6m + 2n - (12n - 10m)\} - m$
$- (7m - 4n)]$

129. Find the numerical values of algebraic expressions, after having made preliminary simplifications ;

(1) N = A − [B − (C − D)] where :
$A = 5x^3 - 4x^2 - 7x + 10$
$B = 3x^3 + 5x^2 - 13x + 2$

$C = -x^3 + 8x - 2x^2 - 9$

$D = x^3 - 7x^2 + 12x - 1$ and $x = \frac{1}{2}$

(2) $N = (2A - 3B) - [3A - (2A + 4B - C)]$ where

$A = 3t^2 - 5tu + 4u^2$;

$B = 2t^2 + tu - u^2$;

$C = 4t^2 - 4tu + 2u^2$ and $t = -0{\cdot}6$

$u = 0{\cdot}8$

130 In the following expressions change the sign before the brackets by the opposite sign without changing the values of the expressions .

(1) $a - (2b - 3a)$ (2) $x - y - (y - x)$

Using equations solve the following problems

131 A log 8·5 *m* long was sawed into three parts, the middle part being greater than the first part by 2 *m* and less than the last by 1·5 *m*. Find the length of each part.

132. The lengths of the sides of a triangle are expressed by successive odd numbers. The greatest side is less than the sum of the other two by 15 cm. Find the perimeter of the triangle.

133. Find the angles of a triangle if its one angle is greater than the second by 25° and the third angle is less than the first by 20°.

134. The difference of two numbers is 72 and the quotient obtained on dividing the one by the other is 4. Find the numbers.

135. The sum of two numbers is 45, and their ratio is 7 : 8. Find the numbers.

136. The speed of a moving steamer down the stream is 18 km per hour and up the stream it is 14 km per hour. Find the speed of the steamer in still water.

137. The area of the island of Madagascar is greater than that of Cuba by 482 thousand km^2 and the area of Cuba is greater than that of Ceylon by 42 thousand km^2. Find the area of each of these islands, if it is known that the area of Madagascar is greater than the sum of the areas of Ceylon and Cuba by 416 thousand km^2.

138. Solve the following equations for x :

(1) $(x + a) + (x + 2a) - (x - 3a) = 8a$
(2) $x^2 - (x + t) - (x^2 - 2x - 3t) = 0$
(3) $(x - a - b) + (2x + 3a + b) = (2a - b) - (2a - 5b)$

Problems and Exercises for Revision

139. Simplify :

$$10 - (6{\cdot}7y + 5y^2) + \{(3y^2 - 1{\cdot}5y + 2{\cdot}3) - (6y^2 + 6y - 1{\cdot}1) + (2y^2 - 0{\cdot}9y + 1{\cdot}3)\}$$

140 Represent the following polynomials as difference of two polynomials :

(1) $4x^3 - 2x - y - y^2$, where
$4x^3 - y$ is the subtrahend

(2) $a^4 + 2a^3 - 9$, where $2a^3$ is the subtrahend.

141. Find the value of the polynomials :

(1) $m + k - p$, if $m = 3a^2 - b^2 + 5c^2$
$k = a^2 - 2b^2 + c^2$
$p = 8a^2 + 4b^2 - c^2$

(2) $x - y + z$, if $x = 10a^2 - 9a + 13$
$y = 2a^2 + 6a - 8$
$z = 5a^2 - 7a - 5$

142. With the help of an equation, solve the problem given below : The perimeter of an isosceles triangle is 48 cm ; the base of the triangle is greater than each side by 3 cm. Find the sides of the triangle.

143. Prove that the sum of any seven successive natural numbers can be divided by 7 without a remainder.

§ 14. Multiplication of Monomials and Polynomials

(*a*) *Use of Powers in Multiplication*

144. Calculate by the simplest method :

(1) $(0{\cdot}25) \times (-0{\cdot}46) \times (-4)$

(2) $\left(-\frac{3}{19}\right) \times (-80) \times 19)$

145. Make operations and find the result .

(1) $c \times c^{11}$ (2) $k^5 \times k \times k^4$

For example .

$$b^3 \times b = b \times b \times b \times b = b^4 = b^{3+1}$$

Hence, for multiplying numbers involving powers of the same base we have simply to add the powers and make the sum as the powers of the common base.

(*b*) *Multiplication of Monomials*

Let us multiply monomial $2xy^2$ by monomial $3x^3y$, *i.e.*, we have to find the value of $2xy^2 \times 3x^3y$

Using commutative and associative laws, we get :

$$2xy^2 \times 3x^3y = (2 \times 3) \times (x \times x^3) \times (y^2 \times y) = 6x^4y^3$$

146. Multiply :

(1) $2a^2$ by $(-a)$ (2) $0{\cdot}3a^2x$ by $2ay$

(3) $(1{\cdot}52)$ by $(-0{\cdot}25)$ (4) $(-3a)$ by $(+2b)$ by $(-c)$

In order to multiply monomials we have to :

(1) **multiply their numerical coefficients,**

(2) **add the exponents of the same base occurring in both cofactors,**

(3) **keep without alteration the exponent of bases which occur only once in the cofactors.**

147. Make operations (Oral) :

(1) $(-b) \times (-b)^{2n-1}$ (2) $(-x)^{2n+1} \times (-x)^{2n+3}$

(3) $(-m) \times (-n) \times 0$ (4) $0 \times (-b)^{k} \times (-b)^{k-1}$

148. Simplify .

(1) $(0{\cdot}06a^4xy) \times (^{+}1{\cdot}5xy^3)$

(2) $(^{+}2{\cdot}4k^2b^4) \times \left(\frac{-1}{2}k^3\right)$

(3) $(^{-}4{\cdot}7bx^3) \times (^{-}4{\cdot}3b^2x)$

149. Multiply :

(1) $(^{-}5x^{m+1})$ by $(^{-}2x^2)$ (2) $(a-b)^5$ by $(a-b)^4$

(3) $-(x-2)^3$ by $(x-2)^7$

150. Simplify ·

(1) $(2a^3)^2 \times (^{-}3a)$ (2) $(^{+}5x^n)^2 \times (^{+}2xy)^2$

(3) $(+x^ny^m)^2 \times (-x^ny^m)^3$

(4) $[-(-a)^2]^2$ (5) $-(^{-}3ab)^2$

(*c*) *Multiplication of a Polynomial by a Monomial*

151. Calculate in two ways :

(1) $(18 + 15) \times 4$

(2) $\left(\frac{5}{19} - \frac{1}{5}\right) \times 190$

152. Find in two ways the area of the rectangle ABCD, shown in the figure given below :

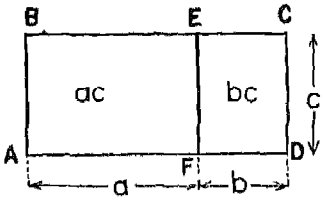


Fig. 2.4

(*i*) Rect. ABCD = Rect. ABEF + Rect. EFDC

$$= BE \times AB + EC \times CD$$

$$= ac + bc$$

(*ii*) Rect ABCD $= \text{BC} \times \text{CD}$

$= (\text{BE} + \text{EC}) \times \text{CD}$

$= (a + b) \times c$

$= ac + bc$

153. Simplify (Oral) .

(1) $(z - 2n) \times k$ (2) $(11a - 7c) \times 9$

(3) $(2a + 3b - 5c) \times 5a$ (4) $(3c - 2b + 7a) \times 4n$

In order to multiply a polynomial by a monomial, it is necessary to multiply each term of the polynomial by the monomial and to add the products thus obtained.

154. Multiply :

(1) $\left(-1\frac{1}{2}p^2 - \frac{3}{4}pq + q^2\right)$ by $(-2pq)$

(2) $\left(1\frac{1}{3}ab\right)$ by $\left(\frac{3}{4}a^2b - 1\frac{1}{2}ab^2 - \frac{5}{6}b^3\right)$

(3) $(2{\cdot}25x^2 - 1{\cdot}5xy + 2{\cdot}5y^2)$ by (-1)

(4) $\left(3a^2 - 5df - 1\frac{1}{2}f^2\right)$ by 0

155. Multiply :

(1) $\left(\frac{1}{8}p^m - \frac{3}{4}p^{11-m}q^1\right)$ by $(-4p^mq^{m-n})$

(2) $-6m^xn^x$ by $\left(\frac{-1}{3}m^{2-x} - \frac{1}{2}n^{1-x}\right)$

156. Simplify :

(1) $a(a + b) + b(a - b)$

(2) $-3(a - b) - 2(a + b) - (3a - 2b) + 5(a - 2b)$

(3) $5(2{\cdot}4 - 0{\cdot}9x + 0{\cdot}16x^2) - 4(-1 + 1{\cdot}5x + 0{\cdot}2x^2)$

(4) $4y - 2(x - 3) - 3[x - 3(4 - 2x) + 8]$

157. Solve the following equations :

(1) $5\left(y + \frac{2}{3}\right) - 4\left(3y - \frac{1}{2}\right) = 3$

(2) $\frac{a-1}{1} + \frac{3a+1}{5} = 6$

(3) $\frac{5(21-5m)}{12} = 14 - 1$

(4) $\frac{3p+41}{2} - \frac{p-3}{5} - \frac{9-2p}{6} = 0$

Solve the problems by working out equations :

158. The length of a rectangle is twice its width. If the width of the rectangle is increased by 3 m, the area of the rectangle will be increased by 24 m^2. Find the original length and width of the rectangle

159. One worker makes 187 tools per day. In 24 days he made 213 tools more than what a second worker made in 25 days. Find the number of tools which can be made by the second worker per day.

160. (1) Change Re. 1 into coins of 5P and 3P so that there would be 26 coins in all.

(2) There are 1000 rabbits and hens at a school farm. All hens and rabbits have 3150 legs. Find the number of rabbits and hens respectively

(*d*) *Multiplication of a Polynomial by a Polynomial*

161 (1) Consider the area of the rectangle ABCD given below and show that

$$(a+b)(c+d) = ac + bc + ad + bd.$$

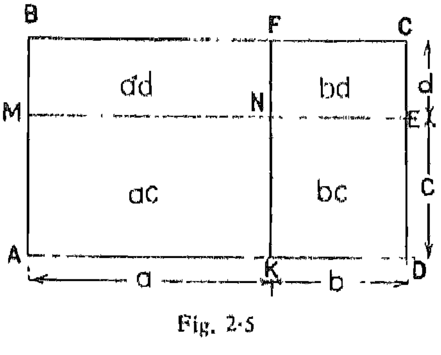


Fig. 2·5

(2) Consider the area of the rectangle ABCD and show that

$$(a + b)(c - d) = ac + bc - bd - ad.$$

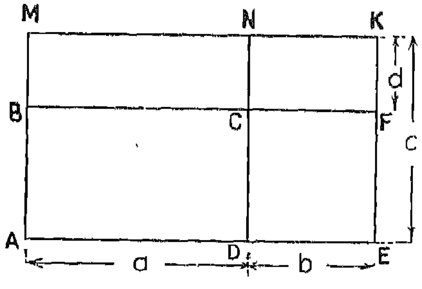


Fig. 2·6

162. Check the following equality :

$(a + b)(m + n) = am + bm + an + bn$, if

No.	1	2	3	4
$a =$	5	10	$\frac{3}{4}$	0·5
$b =$	2	-6	$-\frac{1}{2}$	0·3
$m =$	3	20	4	-5
$n =$	6	-4	2	-8

In order to multiply a polynomial by another polynomial we have to multiply each term of the one polynomial by each term of the other polynomial and then add the terms thus obtained.

163. Find the product :

(1) $(5p + 2q)(9m - 7n)$

(2) $(6a^2 + 5b^2)(2a^2 - 4b^2)$

(3) $(-7x^2 - 8y^2)(- x^2 + 3y^2)$

(4) $(x^2 + 2xy - 5y^2)(2x^2 - 3y^2)$

(5) $x^3 + x^2y + xy^2 + y^3)(x - y)$

(6) $(b^3 - b^2y + by^2 - y^3)(b + y)$

164. Simplify :

(1) $(3a^{n-1} + b^{n-1})(a + b)$

(2) $(a^{x+2} - b^{x-2})(a^2 - b^2)$

If the terms of two polynomials are arranged in descending order of powers, it will be convenient to multiply them through columns.

For example :

$(3x^2 + 11 - 5x) \times (8x - 6 + 2x^2)$ can be put as

$$\begin{array}{rrrrr}
3x^2 & -\ 5x & +\ 11 & & \\
2x^2 & +\ 8x & -\ 6 & & \\
\hline
6x^4 & -\ 10x^3 & +\ 22x^2 & & \\
 & +\ 24x^3 & -\ 40x^2 & +\ 88x & \\
 & & -\ 18x^2 & +\ 30x & -\ 66 \\
\hline
6x^4 & +\ 14x^3 & -\ 36x^2 & +\ 118x & -\ 66 \\
\hline
\end{array}$$

165. Arrange the polynomials suitably and multiply by the column method :

(1) $(15a - 2a^2 - 9)$ by $(-6a + 10 - a^2)$

(2) $(5x + 2x^4 - 3x^2 + 1)$ by $(2 + x^2 - 3x)$

166. Calculate the constant term and the term containing the highest power in the product without actual multiplication :

(1) $(7 + 3a^2 - 5a)(- a + 1 - a^2)$

(2) $(x^6 - 11x^5 + 2{\cdot}3x^7 - 7x + 0{\cdot}75)(1{\cdot}5x^3 - 8x^2 + 6x)$

167. Prove by actual multiplication that,

$$(x + a)(x + b) = x^2 + (a + b)x + ab.$$

This may be used as a formula.

168. Multiply using the formula obtained in problem 167

(1) $(x + 3)(x + 5)$ (2) $(n + 9)(n - 2)$

(3) $(m - 2)(m - 3)$ (4) $(x - 12)(x + 1)$

169. Find the numerical values of the following expressions, after having made the preliminary simplifications ·

$$N = (2x^3 + x^2y + 3xy^2 - y^3)(x + 5y) - (x^3 - 2x^2y$$
$$+ 3xy^2 - 5y^3)(2x - y) \text{ ; if } x = \tfrac{1}{2}, y = 3$$

170. Solve the following equations :

(1) $6x^2 - (2x - 3)(3x + 2) = 2$

(2) $3(x + 1)(x + 2) - (3x - 4)(x + 2) = 36$

171. If one side of a square is increased by 8 m, and the other is decreased by 5 m, the area of the rectangle so formed will be less than that of the original square by 4 m². Find the side of the square.

172. If the length of a rectangle is decreased by 4cm, and the width increased by 5 cm, the area of the rectangle thus formed will is be greater than the area of the original rectangle by 40 cm². Find the area of the rectangle if its width be equal to 15 cm.

173. The product of two consecutive natural numbers is less than the product of the next two consecutive natural numbers by 38. Find the numbers.

174. (1) Take four successive natural numbers. Prove that the product of the two mean numbers is greater than the product of two extreme numbers by 2.

(2) Take four successive odd numbers. Prove that the product of the extreme numbers is less than the product of the mean numbers by 8.

175. The length of a rectangle is twice its width. If we increase the width by 4 m and decrease the length by 5 m, the area of the rectangle will be increased by 16 m². Find the length and the width of the rectangle

176. Simplify the following :

(1) $(2x + 3)(5x - 1) - (4x - 6)(1 - 3x)$

(2) $(a^2 + 2a - 1)(a^3 - 1) - (a^2 - a + 1)(a^2 + 1)$

177. Prove the following equalities :

(1) $2(3a - 4b) - 5[(2a + b) - (a - 2b)]$
$- [3(a - b) - 6(2a - b)] = 10a - 26b$

(2) $a(3a - x) - 2x[(-5a + 1) - (1 - x)]$
$- [x(a - x) - a(2x - a)] = 2a^2 + 10ax - x^2$

178. Solve the equations .

(1) $(3x - 1)(2x - 3) - (6x - 5)(x - 2) + 4 = 0$

(2) $(3y^2 + 7y - 6)(2y - 5) - (2y^2 - 3y + 10)(3y + 4) = 3$

179. The length of a school sports-ground, which has a rectangular form, is equal to 41·5 m and the width is equal to 27·5 m. The external perimeter of the bordering path is 154 m. Find the width of the path if it is uniform for the whole of the ground.

180. Two cylinders start rolling along an inclined plane. The circumference of one of them is 2 dm and that of the other is equal to 3 dm. Find the length of the slope if the first cylinder does 7 rotations more than the second cylinder for the same distance.

15. Raising Polynomials to a Power

Formulas for Short Multiplication

Raising a polynomial to a power means multiplying the polynomial by itself successively as many times as the power to which it is raised.

For example :

$$(2x^2 - x + 1)^2 = (2x^2 - x + 1)(2x^2 - x + 1)$$
$$(3x^2 - b)^3 = (3x^2 - b)(3x^2 - b)(3x^2 - b)$$

There are certain particular cases of these powers which help us in simplifying such expressions.

These particular cases may be treated as laws or formulas.

Study the following cases carefully :

(a) Raising Monomials to Powers

Let us raise x^2 to cube power or to the power three :

$$(x^2)^3 = x^2 \times x^2 \times x^2 = x^{2+2+2} = x^{2\times 3} = x^6$$

By analogy :

$$(b^4)^2 = b^4 \times b^4 = b^{4+4} = b^{4\times 2} = b^8$$

Generally :

$$(a^m)^n = a^m \times a^m \text{......}(n \text{ times}) = a^{m\times n} = a^{mn}$$

When raising a power to a power, exponents are multiplied and the base remains unchanged.

181. Simplify (Orally) :

(1) $(z^5)^k$ (2) $(x^k)^5$

(3) $(y^n)^8$ (4) $(x^{k-1})^4$

182. Solve :

(1) $(-c^6)^2$ (2) $(-c^{2n+1})^2$

(3) $(-y^{2n-1})^{2n}$ (4) $[-(-y)^{2n}]^3$

Let us raise the monomial $3m^2p$ to power 3.

$$\begin{aligned}(3m^2p)^3 &= 3m^2p \times 3m^2p \times 3m^2p\\ &= 3^3 \times (m^2)^3 \times p^3 = 27m^6p^3\end{aligned}$$

By analogy :

$$\begin{aligned}\left(\frac{1}{2}x^2yz^3\right)^2 &= \frac{1}{2}x^2yz^3 \times \frac{1}{2}x^2yz^3\\ &= \left(\frac{1}{2}\right)^2 \times (x^2)^2 \times y^2 \times (z^3)^2\\ &= \frac{1}{4}x^4y^2z^6.\end{aligned}$$

In order to raise a monomial to a power, it is enough to raise each of its factors to the same power and multiply the results so obtained.

183. Obtain the result after raising the monomial to the power indicated over the brackets :

(1) $\left(\frac{1}{3}b^3\right)^2$ (2) $(-0{\cdot}3x^2z)^3$

(3) $-\left(2\frac{1}{2}x^4y^3\right)^3$ (4) $5(-a^3b^2c)^3$

(5) $\left(-3\frac{1}{3}a^nb^2\right)^3$ (6) $4\left(\frac{-2}{3}x^3y^4\right)^3$

184. Simplify :

(1) $(a^{n-1})^3$ (2) $(0{\cdot}2a^{2k-11}x^{10-k})^4$

(3) $(0{\cdot}5x^{k+1}y^{3k})^3$ (4) $(a^3)^{2k-15}$

185. Find which of the following expressions are equal :

(1) $-x^2$ and $(-x)^2$ (2) $-x^3$ and $(-x)^3$

(3) $-(2a)^4$ and $(-2a)^4$ (4) $-(2a)^5$ and $(-2a)^5$

186. Multiply :

(1) $(a^5b^2)^3$ by $(a^2b^3)^2$ (2) $(b^4c^3)^4$ by $(bc^2)^3$

(3) $\left(\frac{1}{9}a^2c^4\right)^2$ by $(3a^6c^2)^6$ (4) $(-15a^4x)^3$ by $\left(-\frac{1}{3}ax^4\right)^4$

(5) $(-12a^5y^3)^4$ by $\left(-\frac{1}{18}a^3y^2\right)^2$

(*b*) *Square of a Binomial* (*Sum of Two Quantities*)

187. Write down the square of the sum of the numbers :

(1) m and k (2) b and c

(3) $-3a$ and x (4) $2a^2b$ and $\frac{1}{2}xy$

Let us square the sum of two numbers x and y.

$$(x + y)^2 = (x + y) \times (x + y)$$
$$= x^2 + xy + xy + y^2$$

Hence :

$$(x + y)^2 = x^2 + 2xy + y^2$$

Square of the sum of two quantities is equal to the sum of squares of the two quantities together with twice the product of those quantities.

188. Calculate the area of the square ABCD, and show that :

$$(x + y)^2 = x^2 + 2xy + y^2$$

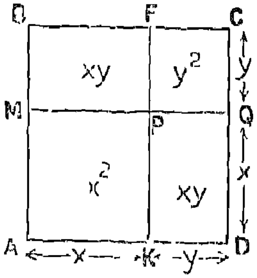


Fig 27

189. Use the formula and find the value of :

(1) $(a + 5)^2$

(2) $(3c + 1)^2$

(3) $(3ax^3 + 8a^3x)^2$

(4) $\left(\frac{a}{4} + \frac{b}{3}\right)^2$

(5) $\left(2m + 1\frac{1}{2}\right)^2$

190. Expand the following expressions with the help of the formula :

(1) $(a^m + b)^2$

(2) $\left(\frac{1}{2}a^{n-1}b^2 + a^{n+1}\right)^2$

191. Find the missing terms in the following equalities :

(1) $(2a + ?)^2 = 4a^2 + ? + 25b^2$

(2) $(? + 10a)^2 = 16b^2 + ? + ?$

192. Form squares of the binomials from the following trinomials and put the remainder together with it as shown in the solved example given at the end of the question.

(1) $a^2 + 6a + 13$ (2) $x^2 + 8x + 21$

(3) $4a^2 + 12ab + 10b^2$ (4) $1 + 10b + b^2$

For example :

$$\begin{aligned} p^2 + 14p + 50 &= p^2 + 14p + 49 + 1 \\ &= (p + 7)^2 + 1 \end{aligned}$$

193. Use the square formula to calculate the value of :

(1) $(103)^2$ (2) $(5{\cdot}1)^2$

194. Solve the following equations :

(1) $x^2 - (x + 2)^2 = 8$

(2) $(2x + 5)^2 - 4x^2 + x = 4$

(3) $(x + 1)^2 - (x - 2)(x + 1) = 3$

(c) *Square of the Difference of Two Quantities*

195. Write down the square of the difference of two quantities :

(1) a and b (2) $2x$ and y

Let us square the difference of two quantities.

$$\begin{aligned} (x - y)^2 &= (x - y) \times (x - y) \\ &= (x^2 - xy - xy + y^2) \end{aligned}$$

Hence .

$$(\mathbf{x} - \mathbf{y})^2 = \mathbf{x}^2 - 2\mathbf{x}\mathbf{y} + \mathbf{y}^2$$

Square of the difference of two quantities is equal to the sum of the squares of the two quantities, minus twice the product of the two quantities.

196. Find the square of (Orally) :

(1) $(c - d)$ (2) $(a - 4)$

(3) $(3a - b)$ (4) $(0{\cdot}1b - 2)$

197. Find the area of the square ABCD and show that :

$$(x - y)^2 = x^2 - 2xy + y^2$$

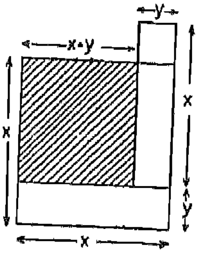


Fig. 2.8

198. Use the formula to expand the following :

(1) $(a^7b^6 - b^5)^2$ (2) $(0{\cdot}2a^5x^3 - 5b^3y)^2$
(3) $(0{\cdot}2y^2 - 0{\cdot}3y^3)^2$ (4) $(2{\cdot}4p^3 - 5q^2)^2$
(5) $(a^m - a)^2$ (6) $(2xy^{p-1} - y^1)^2$

199. Compare the formulas :

$$(x + y)^2 = x^2 + 2xy + y^2$$

and $$(x - y)^2 = x^2 - 2xy + y^2$$

Point out the likeness and the difference between them after writing them as given below :

$$(x \pm y)^2 = x^2 \pm 2xy + y^2$$

200. Simplify :

(1) $(1{\cdot}2x^2y - 0{\cdot}5x^3y^2)^2$

(2) $(-2{\cdot}5m^2n^3 - 0{\cdot}2m^3n^2)^2$

(3) $\left(-1\frac{2}{3}a^{3n} - 1\frac{1}{5}a^{5-3n}\right)^2$

201. What expression should be added to $(a - b)^2$ to obtain $(a + b)^2$?

202. Fill up the gaps to make each of the following expressions a perfect square :

(1) $m^2 - 2mn + ?$ (2) $1 - 2a + ?$
(3) $x^2 - ? + 25p^2$ (4) $? - 12a^2x + 9x^2$

203. Pick out the square of some binomials from the following trinomials :

(1) $x^2 - 20x + 3$ (2) $x^2 - 10x + 27$

(3) $a^2 - 12a - 1$ (4) $9x^2 - 30x + 20$

204. Find the missing terms in the following equalities :

(1) $(? - 3x)^2 = 49b^2 - ? + 9x^2$

(2) $(8ab - ?)^2 = ? - ? + 9b^2$

205. Using the formula $(x - y)^2 = x^2 - 2xy + y^2$, calculate the values of :

(1) $(999)^2$ (2) $(4\cdot9)^2$

206. Simplify the following expressions :

(1) $3(5x - 2y)^2 + (3x - 5y)^2$

(2) $2(7a - 3x)^2 - 3(a - 14x)^2$

(3) $(3a - 5b)^2 + (2a + 9b)^2 - (2a + 3b)(27b - 16a)$

(4) $(4p - 7q)^2 - (3p + 5q)^2 - 2(2p + 3q)(4p - 17q)$

207. Prove the following identities :

(1) $(a - b)^2 = (b - a)^2$

(2) $(-a - b)^2 = (a + b)^2$

(3) $(2p - q)^2 (p + q) - (2p + q)^2 (p - q) = 2q^3$

208. Solve the equations :

(1) $9x^2 - (3x - 5)^2 = 20$

(2) $(13z - 2)^2 - (12z - 5)^2 - (5z + 4)^2 = 19$

d) Product of the Sum and the Difference of Two Quantities :

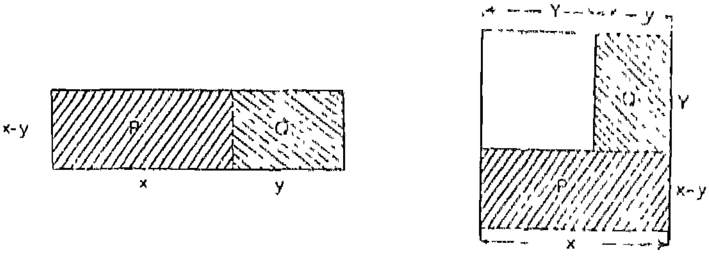
By actual multiplication,

$$(x + y)(x - y) = x^2 + xy - xy - y^2$$
$$= x^2 - y^2$$

Hence .

$$(x + y)(x - y) = x^2 - y^2$$

Explain the above formula with the help of Fig. 2.9.



(a) (b)

Fig 2.9

Thus, **the product of the sum and difference of two quantities is equal to the difference of the squares of these quantities.**

209. Find the product of the following binomials (Oral) :

(1) $(a - c)(c + a)$ (2) $(a - 1)(1 + a)$

(3) $(2a + b)(2a - b)$ (4) $\left(2d - \frac{1}{2}\right)\left(2d + \frac{1}{2}\right)$

(5) $(a^2 + b^2)(b^2 - a^2)$ (6) $\left(\frac{1}{3}x - 0{\cdot}5y\right)\left(\frac{1}{3}x + 0{\cdot}5y\right)$

210. Simplify with the help of the formula given above :

(a) (1) $\left(2\frac{1}{2}x - \frac{3}{4}y\right)\left(2\frac{1}{2}x + \frac{3}{4}y\right)$

(2) $(0{\cdot}02t - 0{\cdot}5u)\ (0{\cdot}5u + {\cdot}02t)$

(3) $\left(2{\cdot}3k + \frac{1}{3}f\right)\left(\frac{1}{3}f - 2{\cdot}3k\right)$

(4) $(a^4 + b^4)\ (a^4 - b^4)$

(b) (1) 38×42 (2) 101×99

(3) $57^2 - 43^2$ (4) $\frac{40^2 - 17^2}{44^2 - 25^2}$

211. Prove the following identities :

(1) $5a^2 - 3\ (a + 1)(a - 1) = 2a^2 + 3$

(2) $(x+y)(x-y)(x^2+y^2)(x^4+y^4) = x^8 - y^8$

(3) $(a+b-c)(a+b+c) = a^2 + 2ab + b^2 - c^2$

212. Simplify :

(1) $(m+1)^2 + 3(m-1)^2 - 5(m+1)(m-1)$

(2) $-(3+x)^2 + 5(1-x)^2 - 3(1-x)(1+x)$

(3) $(2x-3y)(2x+3y)(4x^2+9y^2)$

(4) $\left(\frac{1}{2}x^2 + 2y - xy\right)\left(\frac{1}{2}x^2 - 2y + xy\right)$

213. Solve the following equations :

(1) $(3x+5)(3x-5) - (3x-1)^2 = 10$

(2) $2(2x+1)^2 - 8(x+1)(x-1) = 34$

(3) $(10x-3)^2 - 4(5x-1)(5x+1) = 7$

(e) *Cube of the Sum of Two Quantities*

$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y)\\ &= (x^2+2xy+y^2)(x+y)\\ &= x^3+2x^2y+xy^2+x^2y+2xy^2+y^3\end{aligned}$$

Hence :

$$\begin{aligned}(x+y)^3 &= x^3+3x^2y+3xy^2+y^3\\ &= x^3+y^3+3xy(x+y)\end{aligned}$$

Thus the cube of the sum of two quantities is equal to the sum of the cubes of the two quantities, together with thrice the product of the product of two quantities and their sum.

Expand :

214. (1) $(1+b)^3$ (2) $(x+4y)^3$

(3) $(3ax+1)^3$ (4) $(a+b^3)^3$

215. (1) $(4x^3+5y^2)^3$ (2) $\left(\frac{1}{3}a+\frac{1}{2}b\right)^3$

(3) $(0{\cdot}5x+0{\cdot}1y)^3$ (4) $(0{\cdot}2a^2+0{\cdot}5a)^3$

(5) $\left(10x^3 + \frac{1}{3}y^2\right)^3$ (6) $(b^2 + 1)^3$

(7) $(1 + a^{n+1})^3$ (8) $(0{\cdot}1x^{n-1} + y^{n+1})^3$

216. Find the missing terms in the following equalities :

(1) $(a + ?)^3 = a^3 + 3a^2x + ? + ?$

(2) $(? + 2a)^3 = 1 + ? + ? + ?$

(3) $(2x^3 + ?)^3 = ? + 12x^6y^2 + ? + ?$

217. Prove the following identities :

(1) $a^3 + 3ab\,(a + b) + b^3 = (a + b)^3$

(2) $(2a + b)^3 - (a - b)^2\,(8a + b) = 27a^2b$

218. Solve the equations

(1) $(y + 2)^3 + (2y + 1)^3 - 9y\,(y + 1)^2 = 18$

(2) $(x + 2)(x - 2)^2 - (x + 1)^3 + 5x^2 = 0$

Solve the following problems after having formed an equation :

219. If from the cube of an odd number we subtract the product of three consecutive odd numbers, middle of which is equal to the given number; the remainder is 28. Find these numbers and check your result.

220 Prove that if to the product of two consecutive whole numbers we add the greatest of them, the number obtained is the square of the greatest number.

(f) Cube of the Difference of Two Quantities

$$\begin{aligned}(x - y)^3 &= (x - y)^2\,(x - y)\\ &= (x^2 - 2xy + y)^2(x - y)\\ &= x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3\\ &= x^3 - 3x^2y + 3xy^2 - y^3\\ &= x^3 - y^3 - 3xy\,(x - y)\end{aligned}$$

Hence :

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$
$$= x^3 - y^3 - 3xy(x - y)$$

Thus the cube of the difference of two quantities is equal to the difference of the cubes of the two quantities less by thrice the product of the product of two quantities and their difference.

221. Expand :

(1) $(3 - b)^3$ (2) $(x - 3)^3$
(3) $(2a - 3b)^3$ (4) $(2a^3 - 2b^2)^3$
(5) $\left(\frac{x}{2} - \frac{x}{3}\right)^3$ (6) $\left(0.1b^3 - \frac{1}{2}b^2\right)^3$

222. Compare the formulas :

$$(x + y)^3 = x^3 + 2x^2y + 3xy^2 + y^3$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Observe the likeness and difference between them. We put them together as follows :

$$(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$$

223. Prove the identities :

(1) $(-a - b)^3 = -(a + b)^3$
(2) $(a - b)^3 = -(b - a)^3$
(3) $5(x - y)^3 + 5y(x + y)^2 + x(x - 5y)^2 = 4x^3$

224. Find the missing terms in the following equalities :

(1) $(? - b)^3 = ? - 3z^2b + ? - b^3$
(2) $(5b - ?)^3 = ? - ? + ? - 1$
(3) $(? - a^2)^3 = ? - 3a^2b^4 + ? - ?$

225. Solve the equations .

(1) $(8x - 3)^2 x - (4x - 1)^3 = 7$
2) $(4y - 3)^3 - y(8y - 9)^2 = 0$

(3) $(x-2)^3-(x+1)^3+(3x+2)^3=2$

226 Simplify :

(1) $(a+1)(a^2-a+1)$

(2) $(1+m^2)(1-m^2+m^4)$

(3) $(2a+3)(4a^2-6a+9)$

(4) $\left(n+\frac{1}{2}\right)\left(n^2-n+\frac{1}{4}\right)$

Conclusion :

$$(x+y)(x^2-xy+y^2)=x^3+y^3$$

227. Simplify .

(1) $(b-1)(b^2+b+1)$

(2) $(3a-4)(9a^2+12a+16)$

(3) $\left(\frac{1}{2}a-2b\right)\left(\frac{1}{4}a^2+ab+4b^2\right).$

Conclusion :

$$(x-y)(x^2+xy+y^2)=x^3-y^3$$

228. Use the formulas stated above for simplifying the following :

(1) 19^3 (2) 99^3

(3) $(1{\cdot}01)^3$ (4) $(4{\cdot}8)^3$

229. Simplify the following expressions and find their values :

(1) $2x^3+9-(x+1)(x^2-x+1)$, if $x=\frac{1}{2}$

(2) $(a-1)^3-4a(a+1)(a-1)+3(a-1)(a^2+a+1)$ if $a=-2$

Problems and Exercises for Revision

230. (1) If $x+\frac{1}{x}=4$ find $x^2+\frac{1}{x^2}$

(2) if $x-\frac{1}{x}=3$ find $x^2+\frac{1}{x^2}$

(3) if $x^2 + \frac{1}{x^2} = 7$ find $x + \frac{1}{x}$

(4) if $x^2 + \frac{1}{x^2} = 11$ find $x - \frac{1}{x}$

(5) if $x + \frac{1}{x} = 3$ find $x^4 + \frac{1}{x^4}$

(6) if $x - \frac{1}{x} = 4$ find $x^3 - \frac{1}{x^3}$

231. Prove the following identities

(1) $(a + b + c)(b + c - a) + (b + c - a)(c + a - b) + (c + a - b)(a + b - c) + (a + b - c)(a + b + c) = 4b(a + c)$

(2) $(y + z)^3 - (y - z)^3 = 2z(3y^2 + z^2)$

232. Without actually multiplying the polynomials write down the coefficient of :

(*i*) a^3 in the product $(2a^2 + 1)(a^2 - a + 1)$

(*ii*) y^3 in the product $(1 + 3y)(1 + 2y - 3y^2 + y^3)$

233. (1) Prove that the square of any odd number decreased by 1, is divisible by 8.

(2) Prove that the sum of the cubes of three consecutive natural numbers is divisible by 3.

234. (1) By what number should 2522 be divided to obtain 105 as the quotient and 2 as the remainder ?

(2) By what number should we divide 1239 in order to obtain 103 as the quotient and 3 as the remainder ?

§ 16. Division in Monomials and Polynomials

We have learnt in arithmetic that while dividing a natural number by another natural number we cannot always get a natural number as quotient without a remainder. For example, 17 cannot divided by 5 without a remainder.

In such cases the quotient is written as a fractional number, where the dividend is numerator, and the divisor is the denominator. For example :

$$17 \div 5 = \frac{17}{5},$$

$$\text{and} \quad 3 \div 8 = \frac{3}{8}$$

Introduction of fractional numbers made it possible to divide any number by any other number except zero The same rule of division applies to algebraic expressions also. The quotient is not always a whole algebraic expression. If the quotient after the division of a whole algebraic expression is a whole algebraic expression, it is said that dividend has been divided by the divisor without remainder. Let us take some examples ·

(1) $2ab \div b$

In this case the division is feasible; the quotient is a whole algebraic expression $2a$.

(2) $(2ab + c) \div b$

In this case it is impossible to find such a whole algebraic expression which would be obtained after dividing $(2ab + c)$ by b without a remainder. In this case division without remainder is not possible. The quotient is written as a fractional expression, in which dividend is the numerator. For instance :

$$(2ab + c) \div b = \frac{2ab + c}{b}$$

Later on we will always assume that divisor is not equal to zero, as division by zero is not defined.

That is why if a divisor contains one or more letters such that their product is zero, the division will not be possible. It is necessary to bear this condition in mind when we make division in the algebraic expressions

If the denominator of a given expression is equal to zero for any values of the letters contained in it, such an expression makes no sense. For example, the expression $\frac{a}{b-b}$ makes no sense.

(a) Division of Quantities Involving Power of One and the Same Base

$$2^5 \div 2^2 = (2^3 \times 2^2) \div 2^2 = 2^3 = 2^{5-2}$$

$$3^6 \div 3^4 = (3^4 \times 3^2) \div 3^4 = 3^2 = 3^{6-4}$$

$$a^7 \div a^4 = (a^3 \times a^4) \div a^4 = a^3 = a^{7-4}$$

Generally,

$$a^m \div a^n = a^{m-n} \ (m > n).$$

Let us check it up. We know that $a^{m-n} \times a^n = a^m$. Therefore, $a^m \div a^n = a^{m-n}$.

Thus in division, involving the powers of one and the same base, we subtract the exponent of the divisor from the exponent of the dividend.

If $m = n$, divisor and dividend being equal, the quotient will be equal to 1.

Hence,

$$\begin{aligned} a^m \div a^m &= a^{m-m} \\ &= a^0 \\ &= 1 \end{aligned}$$

235. Apply the operation of division on (Oral) :

(1) $9^8 \div 9^6$ (2) $m^7 \div m^4$

(3) $c^5 \div c^4$ (4) $p^8 \div p^4$

(5) $x^m \div x^m$ (6) $o \div a^0$

236. Divide :

(1) $y^9 \div (-y)$ (2) $-z^7 \div z^5$

(3) $(-p^k) \div (-p^k)$ (4) $a^{m+1} \div a^m$

(5) $x^{m+1} \div x^{n+1}$ (6) $c^{2n+1} \div c^{n+1}$

(b) *Division of Monomials by Monomials*

Let us assume that it is necessary to divide $10a^5b^3c$ by $4a^3b$.

Let us use the rule of dividing the product of several numbers by a number and the rule for dividing a number by the product of several numbers.

$$10a^5b^3c \div 4a^3b = (10 \div 4)(a^5 \div a^3)(b^3 \div b)c$$
$$= 2{\cdot}5a^2b^2c.$$

Having multiplied $4a^3b$ by $2{\cdot}5a^2b^2c$, we will obtain dividend $10a^5b^3c$. So, division is correct.

237. Divide :

(1) $16x^3y^2$ by $4x^2y$ (2) $6b^2cd$ by $3bc$
(3) $15m^2n^3$ by $(-5m^2n^2)$ (4) $18c^4d^2$ by $18c^3d^2$

238. Would it be possible to divide the following monomials without a remainder ?

(1) $5a^2b$ by $4ab$ (2) $15x^2y^2$ by $(-3x^1y)$

Write down the quotient you get on dividing the monomials in the form of fractions.

In order to divide a monomial by a monomial, it is enough

(1) to divide the coefficient of dividend by the coefficient of divisor ;

(2) to subtract the exponents of same factors occurring in the divisor from those occurring in the dividend.

Using this rule it is necessary to bear in mind that

(1) if some letter occurs only in the dividend, then it is retained in the quotient with the same exponent ;

(2) if exponents of some letter in the dividend and the divisor are equal, this letter will not be found in the quotient.

Dividing monomials without a remainder is impossible in the following cases (in the case of natural number exponents only) :

(1) if an exponent of some letter of divisor is greater than exponent of the same letter of the dividend ;

(2) if the divisor contains a letter, which is not contained in the dividend

239. Simplify :

(1) $20m^4n^3 \div (5m^3n^2)$ (2) $8p^4c^3 \div (-2pc)$

(3) $(-12mn) \div 4mn$ (4) $4a^2b^2c \div (-abc)$

(5) $(1{\cdot}2a^8b^3c^4) \div (0{\cdot}3a^5bc^4)$

(6) $\dfrac{-72c^4y^6}{-48c^2y^4}$

240. Divide

(1) $0{\cdot}5a^mb^nc^3$ by $\dfrac{-2}{3}a^2bc$

(2) $-0{\cdot}35b^3 \div 2c^5$ by $0{\cdot}14b^2c^5$

241. Simplify :

(1) $\dfrac{12a^7b^{4n+5}c}{-10a^5b^{4n-1}}$ (2) $\dfrac{(3a^5b^4)^4 \times (2a^3b^2)^5}{(6a^7b^4)^5}$

(3) $(x^5)^4 \times (x^4)^4$ (4) $\dfrac{81^{n+1} \times 3^{n+1}}{27^n \times 9^{n-1}}$

242. Write down the quotient as a fraction :

(1) $a \div 6$ (2) $8 \div c$

(3) $-7b \div (-2c)$ (4) $5ab \div 4pq$

(c) Division of Polynomials by Monomials

Check the justification of the equality given below :

$$(a + b) \div m = \frac{a}{m} + \frac{b}{m}$$

if $a = 36$
$b = 24$
$m = 4$

243. Divide :

(1) $(10a + 5)$ by 5 (2) $(16 - 20b)$ by 4

Let us assume that it is necessary to make the following division .

$$(6a^4b^2 - 7a^3b + 8a^2b^3) \div 2a^2b$$

Having divided each term of the polynomial by $2a^2b$, we will obtain

$$3a^2b - 3{\cdot}5a + 4b^2$$

Let us check it :

$$(3a^2b - 3{\cdot}5a + 4b^2) \times 2a^2b = 6a^4b^2 - 7a^3b + 8a^2b^3$$

In order to divide a polynomial by a monomial, it is necessary to divide each term of this polynomial by the monomial and then *add the results* thus obtained

244. Make the following operations :

(1) $(a^2b + a) \div a$

(2) $(x^3 - ax^2) \div x$

(3) $(9xy^2 - 15x^3y^4) \div (-3xy^2)$

(4) $(-10x^3 - 5x^2 - 20x) \div (-5x)$

(5) $(a^6y - a^4y^3) \div 2a^4$

(6) $(x^2y - xy^2) \div 4xy$

245. Simplify :

(1) $(-4x^2 + 12x^3y^2 - 16x^4y^3) : (-4x^2)$

(2) $(18a^4x^3 + 24a^3x^4 + 6a^2x^5) \div 6a^2x^3$

(3) $(4m^3 + 6m^2 - m) \div \left(\frac{-1}{2}m\right)$

(4) $(-2n^4 - 3n^3 + n^2) \div \frac{1}{3}n$

(5) $(p^4 + 4p^3 - 6p^2 - 8p) \div (0{\cdot}2p)$

(6) $\left(-4m^5n^2 - 4m^4n^5 + \frac{2}{3}m^3n^6\right) \div \frac{2}{3}m^2n^2$

246. Put down the missing monomials in the following equalities .

(1) $\frac{36a - 24a}{?} = 3a - ?$

(2) $\dfrac{144x^7y^5 - ?}{?} = 9x^3y^4 - 2x^2y$

(3) $\dfrac{? - (42b^3c^1)}{28b^2c^2} \qquad 2{\cdot}5b^5c^8 - ?$

247. Simplify :

(1) $(10a - 5) \div 5 - 6(a \quad 1)$
(2) $(6x + 3) \div 3 - (20x - 20) \div 10$
(3) $(a \quad 1) \cdot a - (2a^2 + 4a) \div 2a$
(4) $x(x - 3) - (6x^3 - 12x^2) \div 6x$

248. Prove the following identities :

(1) $(a + b)^3 - (a^2 + ab)^2 \div a - (ab - b^2)^2 \div b = 4ab^2$
(2) $(x - y)^3 - (x^2 - xy)^2 \div x + (xy + y^2)^2 \div y = 4xy^2$

249. Simplify the algebraic expressions and find the result :

(1) $[(5a - 2b)^2 - a^3b^7 \div (-0{\cdot}125a^3b^4)] \div 5a -$
$[(3a - 5b)^2 \quad 4b^2]$ if $a = 2$, $b = 3$

(2) $\left[(3a + 2b)^2 + 5a^5b^{n+1} \div \left(\frac{-1}{8}a^5b^{n-1}\right)\right] \div 9a$
$+ (a - 3b)^2 \quad 4b^2$, if $a \quad 5$; $b \quad 2$

250. Solve the following equations :

(1) $20x^3 \div 4x^2 \quad 14 + x \quad 4$
(2) $8p - 12p^3 \quad 6p^3 + 15 = 45$
(3) $[(y + 4)^3 - (3y + 8)^2] \div y^2 = -1$

251. Divide and check up the results :

(1) $(5a^4b^5 \quad 3a^3b^4 - 15a^4b^3 + 12a^2b^3) \div 3a^2b^3$
(2) $(2{\cdot}8a^4b^5 \quad 0{\cdot}12a^4b^4 - a^3b^5) : (0{\cdot}4a^3b^3)$.

CHAPTER III

Linear Equations with One Unknown

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§1. Set

In this chapter we will describe what we mean by the word 'set'. In daily life you may speak of the collection of printed books you use at school or you may speak of the crowd of people watching a football match. In mathematics we prefer the word 'set' for a collection of objects. The objects, called the *elements* of the set, are often enclosed within the symbol { }.

Example 1

$$\{ 1, 2, 3 \}$$

denotes the set of natural numbers smaller than 4.

Example 2

$$\{ a, b, c, d \}$$

Can verbally be described as the first four letters in the English alphabet.

Sometimes it is rather difficult or time consuming to describe a set by means of a list of the elements within braces. Suppose you have to describe all the natural numbers, smaller than 1000, and that you want to use the braces. We agree upon the following notation .

$$\{ 1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots\ldots, 998, 999 \} \qquad \ldots(1)$$

Where the dots denote the natural numbers from 5 through 997.

1. Write down in a list form the set whose elements can be divided exactly by 6 and are less than 54.
2. Write down in a list form the set of primes between 1 and 52.
3. Write down, using braces, the set of odd integers less than 1000
4. How many elements are there in the set

$$\{ 3, 6, 9, \ldots\ldots\ldots\ldots, 102 \} ?$$

All the sets we have dealt with so far contained only a finite number of elements and are therefore called **finite sets** You have however already dealt with sets, which are not finite, in the previous classes, for example, the set of all whole numbers and the set of all rational numbers. Such sets are called **infinite sets.** An infinite set hascountless elements.

Example 3

The set of whole numbers may be denoted :

$$\{ 0, 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots \}$$

where the dots without a final element denote an infinite set

Example 4

The set of all multiples of 3 may be written as

$$\{ 0, 3, 6, 9, 12 \ldots\ldots\ldots\ldots\ldots\ldots \}$$

5. Which of the following sets are finite sets and which are infinite sets ?
   (*a*) $\{ 1, 2, 3, 4, \ldots\ldots\ldots\ldots 99, 100 \}$
   (*b*) the squares of all positive integers
   (*c*) the set of citizens of India
   (*d*) all positive integers less than 10^{0}
   (*e*) all positive integers greater than 10^{0}

(*f*) the set of all prime numbers

(*g*) the set of rational numbers between $\frac{1}{100}$ and $\frac{2}{100}$

6. Write down within braces the following sets .

(*a*) the set in problem 5*b*

(*b*) the set in problem 5*d*

(*c*) the set in problem 5*c*.

Sets are often represented by capital letters. For example, the set of natural numbers is represented by letter N, the set of whole numbers by W, etc. There are some other standard notations which are commonly used, such as

$$N = \{ 1, 2, 3, \ldots\ldots\ldots\ldots \}$$
$$W = \{ 0, 1, 2, 3, \ldots\ldots\ldots\ldots \}$$
$$I = \{ \ldots\ldots\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\ldots \}$$

(It should be observed that the dots denote infinitely many elements preceding 3).

Q = { all fractions of the form $\frac{a}{b}$, where a, b are integers, and $b \neq 0$ }

(Q is the set of all rational numbers. Observe also that this is a combined use of verbal description and braces.)

Suppose you are asked to write down the elements of the set of all buffalos of green colour. We have to agree that the above sentence describes a set, but a we know, there is no element in it We will agree to call a set without elements, the *empty set*. The empty set is denoted by the Greek letter, ϕ.

Example 5

The set C of whole numbers which are in both the
A = { 1, 3, 5, 7, 9 } and the set B = { 2, 4, 6, 8, 10 } is the
set. Thus C = ϕ.

Example 6

Find the set C of numbers which are in both the sets A = { 0, 1, 3, 5, 7, 9 } and the set B { 0, 2, 4, 6, 8, 10 }. Here we see that C = { 0 }. It is very important to realize that C is not equal to ϕ as it contains an element, namely, 0.

7. Give a verbal description of the set

$$\{ 2, 4, 6, 8, \ldots\ldots\ldots \}$$

8. Write down the set of integers between 31 and 37 which are prime numbers.

9. Enclose the elements of the following sets within braces :

   (*a*) The set of two place integers greater than 99.

   (*b*) The set of two place even integers greater than 98

   (*c*) The set of all the whole numbers between 20 and 40, which are divisible by 4.

   (*d*) The set of all multiples of 5 (greater than zero).

§ 2 Open Statements

Consider the following phrases and sentences :

(*i*) 13 is a prime number

(*ii*) Madras is situated at north of Delhi

(*iii*) Nehru

(*iv*) Open the door

(*v*) $3 \times 7 = 24$

(*vi*) $\frac{1}{2} + \frac{1}{3} < 1.$

We see that 1 and 6 are true sentences, 2 and 5 are false ces. It is however impossible to say that 3 and 4 are true or We agree to call a sentence, of which it is possible to say true or false (but not both true and false), a **statement.**

10. Which of the following phrases and sentences are statements ?

(*i*) a (*ii*) $3 + 6 = 9$ (*iii*) $\frac{3}{4} + \frac{1}{2} < 1$

Example 7

Consider now the following sentence.

"He was a Secretary-General of the United Nations." It is not a statement as we do not know who 'he' is ; on the other hand, it becomes a statement as soon as we choose a name instead of 'he' in a reasonable set of names, say, {Nehru, Kennedy, Hammarskjold}. If we select Hammarskjold we get a true statement, if we choose Kennedy or Nehru we get false statements.

This sentence is an example of an *open statement*, and the set *{Hammarskjold} is the truth set of the open statement* 'He' in the sentence represents a definite, but unspecified person chosen from the set of names in the set {Nehru, Kennedy, Hammarskjold}.

Let us look at a more mathematical example.

Example 8

"$3 + x = 9$" where x should be chosen from the set $\{5, 6, 7, 8\}$ is an open statement.

By substitution we find

$x = 5$	$3 + 5 = 9$	False
$x = 6$	$3 + 6 = 9$	True
$x = 7$	$3 + 7 = 9$	False
$x = 8$	$3 + 8 = 9$	False

The truth set is $\{6\}$. x is a variable, whose concept has already been given to you earlier. Here x represents a definite but unspecified number chosen from a given set of numbers [As from $\{5, 6, 7, 8\}$]. It is often convenient to name the set from which the variable c be chosen as the *domain of the variable*. An element of the do of the variable is called a value of the variable. We often us shorthand symbols.

For example,

Let A = { 1, 2, 3 }. Instead of writing 1 belongs to the set A we write $1 \in A$. The symbol $\in$ is read 'belong(s) to'. We know that $\frac{1}{2}$ does not belong to A,

we write this as $\frac{1}{2} \notin A$

Let an open statement, with a domain D be given. Those elements in D for which the open statement is true, as we know, is called the truth set of the open statement. The truth set in Example 8 can be written as :

$$x \in \{ 5, 6, 7, 8 \text{ such that } 3 + x = 9 \}.$$

If we agree to substitute the phrase 'such that' by a symbol ' : ', we have then

$$\{ x \in D : 3 + x = 9 \} \quad \text{where} \quad D = \{ 5, 6, 7, 8 \}$$

[Note. Two sets are equal if they contain the same elements.]

Example 9

Determine the truth set of the open statement

$$\left\{ x \in Q : x + \frac{1}{2} = 1 \right\} = \left\{ \frac{1}{2} \right\}$$

Read : The set consisting of all x that belong to the rational numbers such that $x + \frac{1}{2} = 1$.

We see that the truth set is equal tothe set which contains the rational number $\frac{1}{2}$.

Note that you have to use braces around the element $\frac{1}{2}$ when you write the truth set.

Example 10

Determine the truth set of the open statement

$$\left\{ x \in N : x + \frac{1}{2} = 1 \right\}$$

We see that there is no such natural number.

Therefore $\left\{x \in N : x + \frac{1}{2} = 1\right\} = \phi$

11. Which of the following sentences and phrases are statements and which are open statements ?

(*a*) Each square is a parallelogram

(*b*) There is a parallelogram that is not a square

(*c*) 7 is a prime

(*d*) India

(*e*) He is a king

(*f*) 2

(*g*) $2 + 3 = 7$

(*h*) $3x = 9$

(*i*) $3x = 9$, where $x \in W$

(*j*) $3 + x = x + 3$ for all $x \in Q$.

12. Determine the truth set for the following open statements :

(*a*) x is the biggest city of Asia, where the domain of the variable is all cities in India.

(*b*) $x \in 3$, where $x \in N$.

§ 3. Equations and Inequations

We will consider only two types of open statements in the rest of the chapter, namely, equations and inequations.

Definition . *An open statement, which contains the sign of equality and an unknown number denoted by a letter, is called an* **equation.**

Definition : *An open statement, that contains some of the symbols* $<, \leqslant, >, \geqslant$ *is called an* **inequation.** (We shall return to this definition later on in this chapter).

We say that we have solved an equation (inequation) if have determined the truth set of the corresponding open ment We call the elements of the truth set *solutions.* (If an equation we also use the name roots for the elements of the truth set). We also say that the elements of the truth set satisfy the equation (inequation).

If the domain of the variable is finite we can find the solutions of an equation (inequation) by substitution as in Example 8, where we verified for each element in the domain if it belonged to the truth set or not.

This simple method is not possible to use if the domain has an infinite number of elements

Example 11

Determine x

$$\{x \in D : x^4 + 3x^2 + x - 3 = 0\}$$

where $D = \{-1, 0\}$.

Solution

$x = -1$ gives : $(-1)^4 + 3(-1)^2 + (-1) - 3 = 0$ true

$x = 0$ gives : $0^4 + 3.0^2 + 0 - 3 = -3$ false

The truth set is $\{-1\}$.

Example 12

Determine x

$$\{x \in D : 4x + 2 < 2x + 1\}$$

where $D = \{-2, -1, 0, 1\}$.

Solution

Substitution gives us

$x = -2$ gives $4(-2) + 2 < 2(-2) + 1$, *i.e.*,
$-8 + 2 < -4 + 1$, *i.e.*, $-6 < -3$ true

$x = -1$ gives : $4(-1) + 2 < 2(-1) + 1$, *i.e.*,
$-2 < -1$ true

$x = 0$ gives : $4.0 + 2 < 2.0 + 1$, *i.e.*, $2 < 1$ false

$x = 1$ gives : $4.1 + 2 < 2.1 + 1$, *i.e.*, $6 < 3$ false.

Answer :

The truth set is $\{ -2, -1 \}$

Before we proceed with solution of equations we remind you of the following property of zero under multiplications.

Form M_1 :

$$a.b = 0 \text{ is equivalent to}$$
$$a = 0 \text{ or } b = 0 \text{ (or both) for all } a \in Q \text{ and } b \in Q$$

Form M_2 :

$$a.c = b.c \text{ is equivalent to } a = b \text{ or } c = 0$$

(or all the three $= 0$) for all $a \in Q$, $b \in Q$ and $c \in Q$.

We often use the symbol $\Leftrightarrow$ instead of the phrase "is equivalent to".

Then :

Form M_1 : $a.b = 0 \Leftrightarrow a = 0$ or $b = 0$ (or both) for all $a \in Q$, $b \in Q$.

Form M_2 : $a.c = b.c \Leftrightarrow a = b$ or $c = 0$ (or all the three $= 0$) for all $a, b, c \in Q$.

In Class VI you have already used the form 2 in examples, as

$$4x = 8.$$
$$4x = 8 \Leftrightarrow 4x = 4 \times 2 \Leftrightarrow x \times 4 = 2 \times 4 \Leftrightarrow x = 2$$

§ 4. Roots of an Equation

Let us consider the following examples each of which is given in different wordings.

Example 13

(*a*) Determine the truth set of

$$\{ x \in Q : 2x - 1 = 0 \}$$

Answer : $\left\{ \frac{1}{2} \right\}$

(*b*) Solve the equation :

$$2x - 1 = 0,$$

where the domain of the variable is the rational numbers or in where $x \in Q$.

Answer : $x = \frac{1}{2}$ (The equation has one root).

Example 14

(*a*) Determine the solution set of

$$\{ x \in Q : (x - 1)(x + 1) = 0 \}$$

Solution

$$(x - 1)(x + 1) = 0$$

$$x = 1 \text{ or } x = -1.$$

Answer : $\{ 1, -1 \}$

(*b*) Solve the equation

$$(x - 1)(x + 1) = 0,$$

where $x \in Q$.

Answer : $x_1 = 1$; $x_2 = -1$ (The equation has two roots).

Example 15

(*a*) Find the truth set of

$$\{ x \in Q : x + 5 = x + 1 \}$$

Answer : The empty set.

(*b*) Solve the equation

$$x + 5 = x + 1,$$

where $x \in Q$.

Answer : There is no such rational number. (The equation has no roots.)

Example 16

(*a*) Find the truth set of

$$\{ x \in Q : (x + 1)^2 = x^2 + 2x + 1 \}$$

Answer : Q (The set of all rational numbers.)

(*b*) Solve the equation :

$$(x+1)^2 = x^2 + 2x + 1,$$

where $x \in Q$.

Answer : The equation is satisfied for all $x \in Q$ (The equation has countless roots).

We saw in the examples that we can formulate the problems in different wordings. We will usually use the forms used in part (*b*) of each of the above examples, both for equations and inequations. We also agree to let the domain of the variable be Q, if not otherwise stated.

In addition to the property expressed by M_1 and M_2 in the solution of Example 14 we will use a similar property of addition.

$$A : a + c = b + c \Leftrightarrow a = b$$

for all $a \in Q$, $b \in Q$ and $c \in Q$.

[**Note 1.** It is obvious that $a = b$ implies that $a + c = b + c$ ($a = b$ means that '*a*' and '*b*' are names of the same number, and as addition is unique it follows that $a + c = b + c$. The other statement that $a + c = b + c$ which implies that $a = b$, is a fundamental property for rational numbers)].

Note 2. It is now quite easy to prove that every equation of the form $a + x = b : x \in Q$; $a, b \in Q$ has at the most one root.

Suppose there were two roots, say x_1 and x_2 that satisfy the equation. We have

$$a + x_1 = b$$

and $$a + x_2 = b.$$

A, gives $a + x_1 = a + x_2 \Leftrightarrow x_1 + a = x_2 + a \Leftrightarrow x_1 = x_2$

That there exists a value of x for which

$$a + x = b$$

depends on the following fundamental property of addition.

For all $a \in Q$ and $b \in Q$ there exists some $x \in Q$ such that

$$a + x = b.$$

We have then, that each equation

$$a + x = b,$$

where $a \in Q$, has one single root in Q. We write

$$x = b - a.$$

Example 17

Solve the following equation in Q :

(*a*) $2x + 5 = x + 4$

(*b*) $x - 3 + 5x = 5x + 5.$

Solution

(*a*) $2x + 5 = x + 4 \Leftrightarrow x + x + 5 = x + 4$

$\overset{\Lambda}{\Leftrightarrow} x + 5 + x = 4 + x \overset{\Lambda}{\Leftrightarrow} x + 5 = {}^-1 + 5$

$\overset{\Lambda}{\Leftrightarrow} x = {}^-1.$

(*b*) $x - 3 + 5x = 5x + 5 \Leftrightarrow x - 3 + 5x = 8 - 3 + 5x$

$\overset{\Lambda}{\Leftrightarrow} x = 8.$

When you are asked to check that your solution is correct, you have to check that the left-hand side of the equation has the same value as the right-hand side in the original equation when the root is substituted for the variable as follows :

(*a*) $x = {}^-1$ gives : L.S. $= 2\,({}^-1) + 5 = 3$
R.S. $= {}^-1 + 4 = 3$
$\therefore\ x = {}^-1$ is a root.

(*b*) $x = 8$ gives : L.S. $= 8 - 3 + 5.8 = 45$
R.S. $= 5.8 + 5 = 45$
$\therefore\ x = 8$ is a root.

We will now solve two more examples, where we make use of the properties M_1, M_2 and Λ.

Example 18

Solve the following equation in Q

$$(x + 4)(x - 2)(x - 1) = 0.$$

Solution

$$(x + 4)(x - 2)(x - 1) = 0$$
$$\Leftrightarrow x + 4 = 0 \text{ or } x - 2 = 0 \text{ or } x - 1 = 0 \quad \ldots M_1$$

Now $x + 4 = 0 \Leftrightarrow x + 4 - 4 = -4 \overset{A}{\Leftrightarrow} x = -4$

and $x - 2 = 0 \Leftrightarrow x - 2 + 2 = 2 \overset{A}{\Leftrightarrow} x = 2$

and $x - 1 = 0 \Leftrightarrow x - 1 + 1 = 1 \overset{A}{\Leftrightarrow} x = 1.$

Answer : $x = -4$ or $x = 2$ or $x = 1$. Note that $x = -4$ and $x = 2$ and $x = 1$ are all roots to the equation.

Example 19

Solve the following equation in Q :

$$(2x - 1)(x + 2) = 0.$$

Solution

$$(2x - 1)(x + 2) = 0 \overset{M_1}{\Leftrightarrow} 2x - 1 = 0 \text{ or } x + 2 = 0$$

$$2x - 1 = 0$$
$$\Leftrightarrow 2x - 1 + 1 = 0 + 1$$
$$\Leftrightarrow 2x = 1$$
$$\Leftrightarrow \frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 1$$
$$\Leftrightarrow x = \frac{1}{2}$$

$$x + 2 = 0$$
$$\Leftrightarrow x + 2 - 2 = 0 - 2$$
$$\Leftrightarrow x = -2$$

Answer : $x = \frac{1}{2}$ or $x = -2$.

Note that both $x = \frac{1}{2}$ and $x = -2$ are roots of the equation.

Hence,

(*i*) **An equation can have only one root.** For example, the equation $2x - 1 = 0$ has only one root $x = \frac{1}{2}$.

(*ii*) **There are equations which can have several roots.** So the equation $(x - 1)(x + 1)(x - 2) = 0$ has three roots. $x_1 = 1$; $x_2 = -1$; $x_3 = 2$.

(*iii*) **Some equations may not have roots at all.** For example, the equation $x + 5 = x + 1$ has no roots at all.

(*iv*) **Finally, an equation can have numberless roots.** The equation $(x + 1)^2 = x^2 + 2x + 1$ has numberless roots because any value of x can satisfy this equation. In such cases we say that the given equation is satisfied identically or is an *identity*.

13. How many roots (solutions) have the following equations ?

(*a*) $x(x + 5) = 0$ (*b*) $2x - 1 = 5$

(*c*) $x^2 + 5 = 9$ (*d*) $4x + 1 = 4x + 3$

(*e*) $5(x - 2) = 5x - 10$

14. Determine which of the following equations are identities (are satisfied identically) :

(*a*) $5x + 3 = 3 + 5x$ (*b*) $4 - x = x$

(*c*) $y = -y$ (*d*) $x + (x + 1) = 2x + 1$

15. Determine which of the following equations are identities and which have no roots at all .

(*a*) $2m + 1 = 2(m + 2)$

(*b*) $5a - 3a - 15 - 6 = 2a - 25$

(*c*) $4z - 3z + 3 + 4 = z + 7$

(*d*) $3x - 12 = 3(x - 4)$

16. Write down the equations with one unknown, which have the following numbers as roots :

(*a*) 4 (*b*) 0 (*c*) -1

17. Explain, why $y = 3$ cannot be a root of the equation:

$$\frac{5}{y-3} = \frac{2}{y-1}.$$

§ 5. Equivalent Equations

Two equations are called equivalent, if each of them has the same roots as the other.

Example 20

The equations $3x + 9 = 15$ and $x + 3 = 5$ are equivalent, as they have the same root $x = 2$.

On the contrary, the equations $x - 3 = 0$ and $(x - 3)(x - 4) = 0$ are not equivalent, as the first of them has only one root $x = 3$ and the other has two roots $x_1 = 3$ and $x_2 = 4$.

The equations which have no roots are also considered equivalent.

So the equations $x + 3 = x + 4$ and $2x - 1 = 2x - 5$ are equivalent.

The equations, which are satisfied identically according to the definition, are also equivalent.

Example 21

The equations

$4y + 3 = 3 + 4y$ and $3z + 6 = (z + 2) \times 3$. are equivalent.

18. Are the following equations equivalent ?

(1) $x + 2 = 0$ and $(x - 2)(x + 2) = 0$

(2) $5x + 1 = 2$ and $10x + 2 = 4$

(3) $2x + 1 = 2x + 2$ and $4x - 5 = 4x - 1$

(4) $x (x - 5) = 0$ and $x - 5 = 0$

19. Write down the equations equivalent to the following equations :

(a) $2x - 1 = 5$

(b) $x - 1 = \frac{2x - 2}{2}$

(c) $2x + 3 = 2x + 1$

20. Check up whether the equation $2x + 1 = x - 3$, is equivalent to an equation, which has the only root $x = -4$

6. Properties of the Equations

We have already learnt how to solve simple equations of the first degree in one unknown, using laws and properties of arithmetical operations At that time we went from one equation to another more simple equation in consecutive order and considered that all these equations were equivalent. In order to convince that it is really so, let us consider the following properties of the equations

Property 1. If we add the same number or the same polynomial to both sides of the equation, then the new equation will be equivalent to the given equation.

Proof :

Let us assume that we have the equation

$$2x + 3 = x - 1.$$

(1) Add to both sides of it the same number 'a' and prove that the equation obtained, $2x + 3 + a = x - 1 + a$ will be equivalent to the given equation

$$2x + 3 = x - 1.$$

Let us assume, that $x = b$ is a root of the equation

$$2x + 3 = x - 1 \quad (\text{in this case } x = b).$$

Then $\quad 2b + 3 = b - 1$

hav

Also $\quad 2b + 3 + a = b - 1 + a,$

one root. nd b are numbers.

It means that $x = b$ is a root of the equation

$$2x + 3 + a = x - 1 + a,$$

then $\quad 2b + 3 + a = b - 1 + a.$

Hence $\quad 2b + 3 + a - a = b - 1 + a - a.$

or $\quad 2b + 3 = b - 1.$

Which shows that $x = b$ is a root of the equation

$$2x + 3 = x - 1.$$

(2) Let us add to both sides of the equation

$$2x + 3 = x - 1$$

the same polynomial, '$ax - c$' and prove that the equation obtained

$$2x + 3 + (ax - c) = (x - 1) + (ax - c)$$

is equivalent to the given equation

$$2x + 3 = x - 1.$$

Let us assume that $x = b$ is a root of the equation

$$2x + 3 = x - 1.$$

Then $\quad 2b + 3 = b - 1.$

Also $\quad 2b + 3 + (ab - c) = b - 1 + (ab - c),$

as $ab - c$ is a concrete number.

It means, that $x = b$ is a root of the equation

$$2x + 3 + (ax - c) = x - 1 + (ax - c).$$

By analogy it is possible to prove that if $x = b$ is a root of the equation

$$2x + 3 + (ax - c) = x - 1 + (ax - c),$$

then $x = b$ is a root of the equation

$$2x + 3 = x - 1.$$

Corollary 1. *If both sides of an equation have the same terms, then it is possible to cancel them.*

Corollary 2. *It is possible to transfer any term of an equation from one side of the equation to the other with signs changed.*

[**Note.** The pupils may be asked to prove these corollaries independe

Below are given solutions of some of the equations proved above with the help of the corollaries and properties.

Example 22

Solve :

$$2x + 5 = x + 4$$

$$2x - x = 4 - 5 \text{ (the corollary 2)}$$

$$x = -1$$

Checking up,

$$2(-1) + 5 = -1 + 4$$

$$3 = 3$$

Answer :

$$x = -1$$

Example 23

Solve for x.

$$x - 3 + 5x = 5x + 5$$

$$x - 3 = 5 \text{ (the corollary 1)}$$

$$x = 5 + 3 \text{ (the corollary 2)}$$

$$x = 8$$

Checking up,

$$8 - 3 + 5 \times 8 = 8 \times 5 + 5$$

$$45 = 45$$

Answer : $x = 8$

21. Use the properties and solve the following equations :

(*a*) $11 + 4x = 3x - 1$
(*b*) $2 - 0{\cdot}5x = 1{\cdot}4 - 1{\cdot}5x$
(*c*) $5 - 6z - 4 = 4 - 7z$
has (*d*) $2\frac{1}{3}x + 1 = 1\frac{1}{2} + 1\frac{1}{3}x$

22. Find the value of x, for which the expression $2x - 1$ and
one root. $x + 1$ have the same numerical values.

23. For what value of y the expression $4y + 3 - 3y$ is equal to 4 ?

Property 2. If both sides of an equation are multiplied by the same number not equal to zero, the new equation will be equivalent to the given equation.

Proof :

Let us assume that the equation

$$2x - 3 = 3x + 1 \text{ is given.}$$

We multiply the sides of this equation by the same number a $(a \neq 0)$ and prove the equation obtained

$$a\,(2x - 3) = a\,(3x + 1)$$

will be equivalent to the given equation

$$2x - 3 = 3x + 1.$$

Let us assume that $x = b$ is a root of the equation

$$2x - 3 = 3x + 1.$$

Then $\quad 2b - 3 = 3b + 1.$

Also $\quad a\,(2b - 3) = a\,(3b + 1).$

It means that $x = b$ is a root of the equation

$$a\,(2x - 3) = a\,(3x + 1).$$

Vice versa. Let us assume that $x = b$ is a root of the equation

$$a\,(2x - 3) = a\,(3x + 1).$$

Then $\quad a\,(2b - 3) = a\,(3b + 1).$

It follows from this that

$$2b - 3 = 3b + 1.$$

$\therefore \quad x = b$ is a root of the equation

$$2x - 3 = 3x + 1.$$

24. Using the property 2, solve the following equations :

(*a*) $-7x = 14$ (*b*) $2\frac{1}{4}y = -4\frac{1}{2}$

(*c*) $-x = -1$ (*d*) $-0{\cdot}65x = 1{\cdot}95$

(*e*) A pupil was asked to solve the following equation :

$$3x - 9 = 2x - 6$$

He gave the following solution :

$$3(x - 3) = 2(x - 3) \text{ or } 3 = 2$$

Find the mistake of the pupil.

25. Solve the following equations :

(*a*) $x - 7 + 8x = 9x - 3 - 4x$

(*b*) $x + 1\frac{1}{2}x + 3 = \frac{2}{3}x + 4 + \frac{5}{6}x - \frac{6}{5}x + \frac{1}{5}$

(*c*) $3 + 2{\cdot}25y + 2{\cdot}6 = 2y + 5 + 0{\cdot}4y$

(*d*) $5z + 0{\cdot}48 - 2{\cdot}35z = 5{\cdot}381 - 2{\cdot}9z + 10{\cdot}42$

(*e*) $2|x| - 1 = |x| + 7$

(*f*) $|x + 3| = -1$

26. Solve the following equations :

(1) $17(2 - 3n) - 5(n + 12) = 8(1 - 7n)$

(2) $(x + 5)(x + 2) - 3(4x - 3) = (x - 5)^2$

(3) $3(z + 1)^2 + (z - 4)^2 = 101 + (z - 3)^2$

(4) $(m + 1)^3 - (m - 1)^3 = 6(m^2 + m + 1)$

(5) $\frac{3a}{2} + \frac{a}{6} - \frac{2a}{9} = 13$

(6) $\frac{1 - 9y}{5} = \frac{19 - 3y}{8}$

§ 7. Generalization

An equation of the form $ax + b = 0$, where x is the unknown number, a (the coefficient of the unknown) is any given number not equal to zero, b (constant term) is also any given number, is called an equation of the first degree in one unknown.

So far only the equations of the first degree in one unknown have been considered.

The equation of the first degree in one unknown has only one root.

Indeed, if we transfer b in the equation $ax + b = 0$, to the right hand side of the equation, we shall obtain a new equation equivalent with respect to the given equation

$$ax = -b.$$

Now divide both sides of this equation by number a (not equal to zero). We shall obtain only one value for x, namely

$$x = \frac{-b}{a}$$

It is possible to give a general rule for solving the equations of the first degree in one unknown :

(1) Reduce the equation to an equivalent equation with whole numbers as coefficients of the variable.

(2) Open the brackets.

(3) Bring the terms containing the unknown on one side of the equation, and the constant terms on the other.

(4) Reduce the similar terms.

(5) If the coefficient of the unknown is not zero, divide each term of the equation by it on both the sides.

§ 8. Solutions of the Problems with the help of Equations

We have already solved problems with the help of the equations according to the following plan :

(1) The unknown value to be found according to the condition of the problem was denoted by the letter x.

(2) With the help of the data we tried to find two expressions using x and the natural numbers.

(3) We equated the two expressions obtained according to the condition of the problem given.

(4) We solved the equation and obtained the value of the unknown letter x.

(5) From this value of x we got the answer.

(6) If several numbers were to be determined according to the condition of the problem , they were also determined as the values of x.

Sometimes it is very useful to work out the special table and to fill it according to the condition of the problem.

Example 24

A person bought 16 big and small balls for children and paid Rs. 24·4 for them. Price of a big ball is Rs. 2·5 and that of the small ball is Rs. 1·2. Find the number of big and small balls separately.

	Number of the balls	Price of one ball	Cost of all the balls
1 Big balls	x	2·5	$2{\cdot}5x$
2. Small balls	$16 - x$	1·2	$1{\cdot}2\,(16 - x)$

According to the condition of the problem :

$$2{\cdot}5x + 1{\cdot}2\,(16 - x) = 24{\cdot}4$$

Solve the following problems with the help of an equation :

27. There are 119 students in three classes of a school. The number of the students in the first class is more than the number of the students in the second class by 4 and is less than the number of the students in the third class by 3. How many students are there in each class ?

28. There are 121 students in 3 classes. In the first class the number of students is less by 4 than that in the second class and is more by 3 than that in the third class. How many students are then in each class ?

29. The weight of the rear wagon of an engine, when it is full, is 122·8 tonnes. It contains water more by 32 tonnes than the coal and the weight of the empty wagon is more by 45·8 tonnes than the weight of the coal. Find the weight of water, coal and the wagon separately.

30. The angle A of a triangle ABC is equal to the sum of the other two angles. Find each angle of the triangle if the ratio of the angles B and C is 4 : 5.

31. Find the angles of a triangle if they are in the ratio of 2 : 7 : 11.

32. The base of an isosceles triangle is equal to $\frac{3}{4}$ of its either side. Find the sides of the triangle, if its perimeter is equal to 22 cm.

33. The weight of grain in one store is twice than that in the other. If we take out 750 tonnes of grain from the first store and 350 tonnes from the second, the amount of grain in the two stores will be the same. How many tonnes of grain does each store contain ?

34. The amount of coal in one stall is twice than that in the second one. If we add 80 tonnes of coal to the first and 145 tonnes to the second, the amount of coal in the two stalls will be the same. Find the amount of coal in each stall.

35. The volume of petrol in a tank is twice than that in the other. If we draw out 25 litres from the first and add it to the other, the volumes of the petrol in each of the two tanks will be the same. Find the volume of petrol in each tank.

36. A father is 40 years old and his son is 16 years old. After how many years father's age will be twice that of his son ?

37. (*i*) If $2\frac{1}{2}$ is subtracted from a number we obtain 3·15. Find the number.

(*ii*) $\frac{2}{5}$ of a number when increased by 1·2, is equal to 7·4. Find the number.

(*iii*) The difference between 3 times of a number and 0·011 is equal to 7·3. Find this number.

38. (*i*) The sum of 3 successive integers is equal to 120. Find these numbers.

(*ii*) The sum of 4 successive numbers is equal to 30. Find these numbers.

39. A number consists of two digits. The digit in the tenth place is 3 times that in the unit place. If the digits are reversed, the new number will be 36 less than the original number. Find the original number.

40. The sum of the digits of a two digit number is 11. If we add 63 to this number, the digits are reversed. Find the number.

41. The sum of the digits of a two digit number is 12. If the digits are reversed the new number formed will be greater by 18 than the original number. Find the original number.

42. The sum of two numbers is 2490. Find the numbers if 6·5 percent of one is equal to 8·5 percent of the other.

43. The distance between two cities is 340 km. Two electric trains start simultaneously from these cities to meet each other. The speed of one of them is greater by 5 km/hr than that of the other. Find the speed per hour of each train if the distance between them is 30 km after 2 hours of their start.

44. Distance between two stations A and B is 230 km. Two motor cyclists start simultaneously from A and B in the opposite directions. After 3 hours the distance between them is 20 km. Find the speed of each cyclist, if the speed of one of them is less by 10 km/hr than that of the other

45. At 4.30 A.M. a helicopter started from town A to reach town B with a speed of 250 km per hour. Having reached town B it stopped for 30 minutes and then came back to A at 11.45 A.M. with a speed of 200 km per hour. Find the distance between A and B.

46. A farm was to be ploughed in 14 days. If 20 ha of land is ploughed more daily, the work will be finished in 10 days Find the area of the land which can be ploughed daily and also the area of the farm.

47. The distance between two ports is covered by a steamer in 2 hrs and 30 mts. If the speed of the steamer is decreased by 6 km per hour, it covers the same distance in 3 hrs and 15 mts. Find the speed of the steamer and the distance between the ports.

48. A postman takes 35 mts to deliver mail from place A to place B. On return he increases his speed by 0·6 km per hour and takes only 30 mts to reach A. Find the distance between A and B and also the two speeds of the postman.

49. A steamer goes down the stream and covers the distance between two ports in 4 hours, and covers the same distance up the stream in 5 hours. Find the distance between the ports, if the speed of the stream is 2 km per hour.

50. A helicopter can cover the distance between two towns in 5 hours 30 minutes while going in favour of the wind ; but against the wind, it covers the same distance in 6 hours. Find the distance between the towns and the speed of the helicopter in calm weather, if the speed of the wind is 10 km per hour.

51. A work of manufacting engines is to be finished in 15 days. If the workers engaged at it increase the rate of their work to prepare two engines more a day, they can manufacture 6 engines more than planned, 2 days before the schedule. How many engines are to be manufactured.

52. A power station is to be built by a group of labourers. They dug 860 cu m of earth daily according to the plan. If the speed of the work is increased by 20 per cent, the work will be completed 2 days earlier How many days will the work take according to the plan ?

53. The weight of an iron-piece and the weight of a copper piece are together equal to 1280 g The volume of the copper piece is twice that of the iron-piece. Find the volume of each piece, if the weight of 1 cu cm of iron is 7·8 g and that of copper is 8·9 g.

54. While solving a problem we got the following equation :

$$5x = (x - 10)$$

Write down the problem relating to the movements of two cars corresponding to this equation.

§ 9. Compound Interest

Example 25

A man lends Rs. 1000 to a Financing Company at the rate of 10 per cent per year. What interest does he get after one year ? What will be the amount then ? At the end of the year he decides to deposit the whole sum (amount after 1 year) for another year. What amount of interest does he get at the end of (*a*) second year, (*b*) two year ?

$$\text{Interest after one year} = \frac{1000 \times 1 \times 10}{100} = \text{Rs. } 100$$

$$\therefore \quad \text{Amount after one year} = \text{Rs. } 1000 + \text{Rs. } 100$$
$$= \text{Rs. } 1100$$

Now again he deposits Rs. 1100 in the same company. What do you notice here ? The amount at the end of first year becomes principal for the second year.

$$\therefore \text{ Interest at the end of IInd year} = \text{Rs. } \frac{1100 \times 1 \times 10}{100}$$
$$= \text{Rs. } 110$$

and interest after 2 years = Rs. (100 + 110) = Rs. 210.

The interest for the second year is more than that for the first year ? Can you tell, why ?

It is clear that in the second case we have calculated interest on Rs. 1100, which is equal to Rs. 1000 (Principal in the beginning) + Rs. 100 (Interest for the first year). So interest on interest has been calculated during the second year. This type of interest is called '*Compound Interest*'.

In the case of compound interest, the calculation of the interest is periodic. After a certain fixed period the simple interest is calculated for that period and is added to the principal. For the next period that amount becomes the *Principal.* Generally, this period is not fixed. But with banks, post offices and other similar bodies this period is generally six months. To make the period clear the phrase *payable half-yearly or annually*, etc., is used.

Example 26

A man deposited Rs. 5000 in savings bank account. How much will it be after one year if the rate of interest is 4 per cent and interest is payable half-yearly.

$$\text{The interest for first half-year} = \frac{\text{Rs. } 5000 \times 1 \times 4}{2 \times 100}$$
$$= \text{Rs. } 100$$

The principal after 6 months = Rs. 5100

$$\therefore \text{ Interest for second half-year } = \frac{5100 \times 1 \times 4}{2 \times 100} = \text{Rs. } 102$$

∴ After one year the amount will be Rs. 5000 + Rs. 100 + Rs. 102 = Rs. 5202, and the compound interest will be Rs. 100 + Rs. 102 = Rs 202.

Example 27

Find the amount of Rs. 500 lent at compound interest at 5 per cent per annum for two years if the interest is payable annually.

$$\text{Interest on Rs. 500 for 1 year} = \frac{(500 \times 1 \times 5)}{100}$$

$$\therefore \text{ Amount after 1 year} = \left[500 + \frac{500 \times 5}{100}\right]$$

$$= \text{Rs. } 500\left(1 + \frac{5}{100}\right)$$

Next we have to find the interest for second year.

$$\text{S.I. on } 500\left(1 + \frac{5}{100}\right) \text{ for 1 yr.}$$

$$= \frac{500\left(1 + \frac{5}{100}\right) \times 1 \times 5}{100}$$

$$\therefore \text{ Amount after 2 years} = \text{P} + \text{Interest}$$

$$= \text{Rs. } \left[500\left(1 + \frac{5}{100}\right) + \frac{500\left(1 + \frac{5}{100}\right) \times 5}{100}\right]$$

$$= \text{Rs. } 500\left(1 + \frac{5}{100}\right) \times \left[1 + \frac{5}{100}\right]$$

$$= \text{Rs. } 500\left(1 + \frac{5}{100}\right)^2$$

$$= \text{Rs. } 500 \times \frac{21}{20} \times \frac{21}{20}$$

$$= \text{Rs. } \frac{2205}{4} = \text{Rs. } 551{\cdot}25$$

Let us develop a formula for finding the amount at compound interest when the principal is P, rate is r and time is n years.

Proceeding as in the above example :

Interest on P for 1 year $= \frac{P \times r \times 1}{100} = \frac{P \times r}{100}$

$\therefore$ Amount after 1 year $(A_1) = P + \frac{P \times r}{100}$

$$= P\left(1 + \frac{r}{100}\right)$$

Now for the second year A_1 becomes principal, so interest on A_1 in the second year

$$= \frac{A_1 \times r \times 1}{100} = \frac{A_1 \times r}{100}$$

$$= A_1 \times \frac{r}{100} = P\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

(Putting the value of A_1)

$\therefore$ Amount after second year or A_2

$$= A_1 + P\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$= P\left(1 + \frac{r}{100}\right) + P\left(1 + \frac{r}{100}\right) \times \frac{r}{100}$$

$$= P\left(1 + \frac{r}{100}\right)\left(1 + \frac{r}{100}\right)$$

$$= P\left(1 + \frac{r}{100}\right)^2$$

Similarly after third year, A_3

$$= P\left(1 + \frac{r}{100}\right)^3$$

In general A

$$= P\left(1 + \frac{r}{100}\right)^n,$$

where n denotes the number of complete years.

$\therefore$ Compound Interest

$$= P\left(1+\frac{r}{100}\right)^n - P$$

$$= P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$$

How will the formula change if the interest is payable half-yearly ?

In this case for r we will write $\frac{r}{2}$ because the interest after half-year will be $\frac{r}{2}$ and for n we will write $2n$ because the number of complete half-years will be $2n$

$$\therefore \quad A = P\left(1+\frac{r}{200}\right)^{2n}$$

Compare the Compound Interest with Simple Interest. On comparison you will notice that :

(*i*) The compound interest and simple interest on the same sum and at the same rate remain the same for the first payable period. So also are their amounts.

(*ii*) The compound interest increases in the subsequent periods whereas the simple interest remains uniform for each period. Therefore the compound interest is always greater than the simple interest after the first payable period.

(*iii*) Hence the amount at compound interest after the first payable period is always greater than the amount at simple interest for the same period at the same rate.

(*iv*) Out of the two amounts for two periods one which is smaller can be treated as principal and the greater as the amount of that principal for the period equal to the difference of two periods at compound interest.

55. Use the formula $A = P\left(1 + \frac{r}{100}\right)^n$ to find the amount and the compound interest when :

(*i*) $P =$ Rs 500, $r = 5\%$, $n = 2$ yrs. interest payable annually.

(*ii*) $P =$ Rs 800, $r = 10\%$, $n = 3$ yrs. interest payable annually.

(*iii*) $P =$ Rs 2000, $r = 8\%$, $n = 1$ yr. interest payable half-yearly.

(*iv*) $P =$ Rs 1500, $r = 4\%$, $n = 1$ yr. interest payable half-yearly.

56. Find the value of the missing term in the formula

$$A = P\left(1 + \frac{r}{100}\right)^n \text{ when}$$

(*i*) $A = 676$, $P = 625$, $r = 4\%$ interest payable annually.

(*ii*) $A = 1210$, $r = 10\%$, $n = 2$ years interest payable annually.

(*iii*) $P = 1000$, $r = 4\%$, $n = 1$ year interest payable half yearly.

(*iv*) $A = 520$, $P = 500$, $n = 1$ year interest payable annually.

57. A man lends Rs. 1000·00 at 10·2 per cent compound interest payable annually ; and another man lends the same sum at 10 per cent compound inetrest but payable half-yearly. Who will be gainer at the end of one year and by how much ?

58. Janki borrowed Rs. 1600 at 5½ per cent C.I. payable yearly whereas Shanker borrowed the same amount at 5 per cent C.I. payable half-yearly. After 1 year who will pay more interest and by how much ?

59. Suresh deposited Rs 1250 in his saving account. Find out the compound interest earned by him after 3 years if the rate of interest be 4 per cent per annum and the interest is payable yearly.

60. Bimla borrowed Rs 400 at compound interest payable annually for buying a sewing machine. She paid back the money together with interest after 2 years. How much money did she pay if the rate was 5 per cent per annum ?

61. The Central Bank pays 4 per cent per annum compound interest on 'Home Savings Deposits' and has a system of adding interest to the principal after every six months. If I deposit Rs 1000 now, how much interest shall I earn after 1 year.

62. Using $A = P\left(1 + \frac{r}{100}\right)^n$ write down an expression for finding C.I. and calculate it when $P =$ Rs. 625, $r = 4$ percent per annum and $n = 2$ years, interest being calculated annually.

63. Fill in the blanks by placing suitable substitute :

(*i*) $\ldots\ldots\ldots\ldots = P\left(1 + \frac{r}{100}\right)^n$

(*ii*) $A = P\left(1 + \frac{r}{100}\right)^{\ldots\ldots\ldots\ldots}$

(*iii*) $\text{C.I.} = A - \ldots\ldots\ldots\ldots$

(*iv*) $\text{C.I.} = P\left(1 + \frac{r}{100}\right)^{\ldots\ldots} - \ldots\ldots\ldots$

(*v*) $A = \ldots\left(1 + \frac{r}{100}\right)^n$

(*iv*) $A = P\left(1 + \frac{\ldots\ldots}{100}\right)^n$

Example 28

I lent Rs. 3200 to my friend, Bhagwati, for 3 years at $2\frac{1}{2}$ per cent per annum simple interest. How much more should I have gained, had I lent it at C.I. payable annually (rate and time being same).

S.I. on Rs. 3200 for 3 years at $2\frac{1}{2}$ per cent

$$= \frac{3200 \times 3 \times 5}{100 \times 2} = \text{Rs. } 240$$

Amount on Rs. 3200 for 3 years at $2\frac{1}{2}$ per cent at C.I.

$$= 3200\left(1 + 2\tfrac{1}{2}/100\right)^3$$

$$= 3200\left(1 + \frac{5}{200}\right)^3$$

$$= 3200\left(1 + \frac{1}{40}\right)^3.$$

$$= 3200 \times \frac{41}{40} \times \frac{41}{40} \times \frac{41}{40} = \frac{68921}{20}$$

$$= \text{Rs. } 3446{\cdot}05$$

$$\therefore \quad \text{C.I.} = \text{A} - \text{P} = 3446{\cdot}05 - 3200$$

$$= \text{Rs. } 246{\cdot}05$$

Hence gain $= 246{\cdot}05 - 240 = \text{Rs. } 6{\cdot}05$

64. A finance company pays 12 per cent simple interest to all the depositors but in turn charges the same rate at C.I. payable yearly from the borrowers. On a certain transaction the company gained Rs. 9 over a period of 2 years. Find the amount of the transaction.

65. Kishan deposited Rs. x in his savings account from where he got C.I. at 4 per cent per annum for a period of 2 years.

Had the interest been simple he would have received Rs. 8 less. Find x.

66. Amar lent Rs. P to 'Globe Financers' at 10 per cent C.I. payable annually for a period of 1 year. The Company lent it further to some needy man at the same rate of interest payable half-yearly thus gaining Rs Y. Find an expression for Y. If Y = 40, find the value of P.

67. While solving a question on C.I. payable yearly a student got the equation $A = P \times \frac{26}{25} \times \frac{26}{25}$. Guess the rate per cent and the time for which the student was calculating C.I. Also find A when P = Rs. 625.

68. Express P in terms of A from the formula

$$A = P \times \frac{41}{40} \times \frac{41}{40} \times \frac{41}{40} \text{ and find it}$$

when $A = 2825761$

69. Solve for n

$$605 = 500\left(1+\frac{10}{100}\right)^n$$

70. A man deposited some money with a finance company which amounted to Rs. 1210 after 2 years. He did not withdraw the money for another 2 years and got Rs. 1464, 10 paise at the end of 4 years. Find the rate of interest calculated, if the compound interest was payable yearly.

71. Ram borrowed Rs. 8000 at 5 per cent C.I. payable annually whereas his friend Vijay borrowed Rs. 8200 at 5 per cent S.I. for purchasing a plot of land After 3 years both of them paid back their loans together with their interests. Who paid more interest and by how much ?

72 Khairati deposited Rs. 2000 with a finance company at 10 per cent C.I. payable yearly whereas his cousin Shyam deposited

Rs. 2200 to another company at 10 per cent S.I After 2 years they got their deposits back along with the respective interests. Who will earn more interest and by how much ?

73. Dayal bought a housing plot on annual instalment basis. He paid Rs. 500 in the beginning, Rs. 425, one year after and Rs. 289, two years later. What would have been the cash value of the plot, had the money been paid in the beginning, compound interest being calculated at $6\frac{1}{4}$ per cent per annum ?

74. Find the compound interest on Rs. 18750 for 2 years if the rate of interest be 4 per cent for the first year and 5 per cent for the second year.

75. Suresh had a fixed deposit of Rs. X in a bank which amounted to Rs. 4840 after 2 years at 10 per cent C I. payable yearly. Find X. For how long should he keep the money there so that it may amount to Rs. 5856·40 ?

§ 10. Inequations

Let us consider the following definition.

Definition : If $a \in Q$ and $b \in Q$, then

$$a < b \text{ if and only if } (b - a) \in Q_+$$

where Q_+ denotes the positive rational numbers.

Example 29

$$\frac{1}{3} < \frac{1}{2} \text{ as } \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \in Q_+.$$

We also remind you of the following denotations :

$a \leqslant b$ is written for $a < b$ or $a = b$

$a \geqslant b$ is written for $a > b$ or $a = b$

Definition : An open statement, that contains some of the symbols $<$, $\leqslant$, $>$, $\geqslant$, is called an inequation.

Example 30

Solve the inequation

$$2x\,(x + 1) < 3x + 6, \text{ where } x \in D = \{\,0,\ 1,\ 2,\ 3\,\}$$

Solution

As there are only a small number of elements in D we solve the inequation by the method of substitution :

Values of x	Statement	True or false
0	$0 < 6$	True
1	$4 < 9$	True
2	$12 < 12$	False
3	$24 < 15$	False

Answer

The truth set is $\{\,0,\ 1\,\}$

In order to solve inequations, where the domain of the variable is a 'big' set we have to use properties of inequalities an analogous to A and M_2, above,

$$\text{I.A} : a < b \Leftrightarrow a + c < b + c$$

for all $a \in Q$, $b \in Q$ and $c \in Q$

Example 31

For which x in Q is

$$5 - 2x < -\,3x + 2 \text{ true ?}$$

Solution

$$5 - 2x < -3x + 2$$

$$\overset{\text{I A}}{\Leftrightarrow} 5 - 2x + 3x < -3x + 2 + 3x$$

$$\Leftrightarrow 5 + x < 2$$

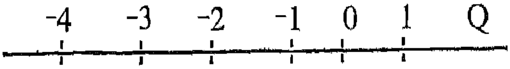
$$\Leftrightarrow x + 5 < -3 + 5$$

$$\overset{\text{I.A}}{\Leftrightarrow} x < {}^{-}3$$

Answer

$$\{ x \in \mathrm{Q} : 5 - 2x < -3x + 2 \} = \{ x \in \mathrm{Q} : x < {}^{-}3 \}$$

The solution set can be illustrated on the number line as follows .



The second property can be stated as follows :

$$\text{I.M} : a < b \Leftrightarrow a.c < b.c$$

for all $a \in \mathrm{Q}$, $b \in \mathrm{Q}$ and $c \in \mathrm{Q}_+$, where Q_+ denotes the set of positive rational numbers, and

$$a < b \Leftrightarrow a.c > b.c$$

for all $a \in \mathrm{Q}$, $b \in \mathrm{Q}$ and $c \in \mathrm{Q}_-$, where Q_- denotes the set of negative rational numbers.

That it is really necessary to distinguish between the case $c \in \mathrm{Q}_+$ and $c \in \mathrm{Q}_-$, will be realised when you look at the example below.

Example 32

$${}^{-}2 > {}^{-}5 \Leftrightarrow ({}^{-}2)\,(4) > ({}^{-}5)\,(4) \text{ but}$$

$${}^{-}2 > {}^{-}5 \Leftrightarrow ({}^{-}2)\,({}^{-}4) < ({}^{-}5)\,({}^{-}4)$$

Example 33

Solve the following inequation in Q

$$\frac{x}{5} + \frac{1}{4} < 0$$

Solution

$$\frac{x}{5} + \frac{1}{4} < 0$$

$$\Leftrightarrow \frac{x}{5} + \frac{1}{4} + \left(\frac{-1}{4}\right) < \frac{-1}{4}$$

$$\Leftrightarrow \frac{x}{5} < \frac{-1}{4}$$

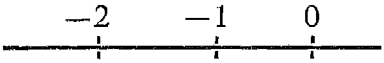
$$\Leftrightarrow \frac{x}{5} \times 5 < \frac{-1}{4} \times 5$$

$$\Leftrightarrow x < \frac{-5}{4}$$

Answer :

$$\left\{x \in Q : \frac{x}{5} + \frac{1}{4} < 0\right\} = \left\{x < \frac{-5}{4}\right\}$$

Geometric illustration :



Example 34

Find the truth set of the open statement

$$\{x \in Q \,.\, 1 + x^2 < 2x\}$$

Solution

$$1 + x^2 < 2x$$

I A.

$$\Leftrightarrow 1 + x^2 - 2x < 2x - 2x$$

$$\Leftrightarrow 1 + x^2 - 2x < 0$$

$$\Leftrightarrow (1 - x)^2 < 0$$

which is impossible as the square of a rational number is always $\geqslant 0$.

Answer .

$$\{ x \in Q : 1 + x^2 < 2x \} = \phi \text{ (the empty set).}$$

Determine the truth sets and illustrate on the number line (when possible).

77. $\{x \in Z : 0 < x\}$

78. $|x| < 0$, where $x \in Q$

79. $2x + 1 < 5$, where $x \in Q$

80. $|x| < \frac{1}{2}$, where $x \in Q$

81. $|x| > {}^{-}1$, where $x \in Q$

82. $2x + 4 < 3x + 2$, where $x \in 1$

83. $3x + 8 < 7x + 10$, where $x \in Q$

84. $\frac{1}{2}x - \frac{1}{3} < \frac{1}{4}x - \frac{1}{5}$, where $x \in Q$

85. $\left\{ x \in Q : 2 - \frac{3}{4}x < 3x + \frac{1}{2} \right\}$

86. $\{ x \in Q : 6 - x < 5 - x \}$

87. $\{ x \in Q \,.\, 2x^2 < x^2 \}$

88. $2x + 4 < 3x + 2$, where $x \in I$

89. $2x < 1 + x$, where $x \in I$

90. $1 - \frac{x}{2} < x$ and $x < 4 - x$,

where $x \in Q$

CHAPTER IV

Factorization of Polynomials

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1. Simplify using factorization method :

   (*a*) $596 \times 23 + 404 \times 23$

   (*b*) $5{\cdot}4 \times 2{\cdot}123 - 0{\cdot}123 \times 5{\cdot}4$

   (*c*) $24\frac{7}{8} \times 1\frac{1}{4} - 4\frac{7}{8} \times 1\frac{1}{4}$

Suppose we have to find the product of two algebraic expressions, for instance x and $a - b + c$.

As a result of multiplication we shall obtain :

$$x(a - b + c) = ax - bx + cx$$

We can write the equality in the above form using the distributive law :

$$ax - bx + cx = x(a - b + c)$$

It means that it is possible to represent the polynomial $ax - bx + cx$ as a product of two factors, one of which is x and the other $(a - b + c)$.

To factorize a polynomial means to transform it identically into the product of two or more factors (polynomials or monomials).

Factorization of algebraic expressions is like the factorization of the natural numbers in arithmetic. While making operations with algebraic fractions, calculating numerical values of algebraic expressions, solving some equations etc, we make use of factorization.

Different methods of factorization are considered below.

§ 1. Taking Out the Common Multiplier

2. Compute by the shortest method :

(*a*) $23{\cdot}4 \times 8 + 56{\cdot}6 \times 8$

(*b*) $34 \times 1{\cdot}78 + 25 \times 1{\cdot}78 + 41 \times 1{\cdot}78$

Let us assume that the polynomial $6a^2 - 8ab + 4a$ is given. We can see that $2a$ is the common multiplier for all terms of the polynomial

It is possible to write down the polynomial $6a^2 - 8ab + 4a$ as $2a \times 3a - 2a \times 4b + 2a \times 2$. If we use distributive law of multiplication, we shall obtain $6a^2 - 8ab + 4a = 2a \times 3a - 2a \times 4b + 2a \times 2 = 2a\,(3a - 4b + 2)$. We see that we can represent the polynomial as the product of two factors ; the monomial $2a$ and the polynomial $3a - 4b + 2$.

Thus in order to factorize a polynomial by taking out the common multiplier it is necessary :

(1) to determine this common multiplier ;

(2) to divide the given polynomial by it ;

(3) to write down the common multiplier and the quotient obtained into separate brackets with signs of multiplication between the brackets

Let us consider an example

In the polynomial $12x^4y^2 - 18x^3y^2 + 24x^2yz^3$

(*a*) 6 is the common multiplier for the coefficients ;

(*b*) the first letter x in the first term of the polynomial is of the 4th degree, in the second term it is of the 3rd degree, and in the third term it is of the 2nd degree. Therefore, the polynomial is divisible by x^2.

(*c*) by analogy we can say that the given polynomial is divisible by y ;

(*d*) we may not consider the letter z, because in the two terms of the polynomial this letter does not occur. And so the monomial $6x^2y$ is the common multiplier for the given polynomial.

Then $12x^4y^2 - 18x^3y^2 + 24x^2yz^3 = 6x^2y\,(2x^2y - 3xy + 4z^3)$

3. Find the divisors of the following polynomials :

(*i*) $3a + 3b$ (*ii*) $6nk - 18np$

(*iii*) $9x^2 - 18x^3$ (*iv*) $7by^2 - 14b^2y^2 + 112y^3$

4. Factorize :

(*i*) $2x + 2y$ (*ii*) $ax - ay$ (*iii*) $-2a - 5ab$

Let us consider an example ;
Factorize the polynomial :

$$10m^3p - 6mp^2 + 2m.$$

$2m$ is common multiplier for this polynomial

Therefore $10m^3p - 6mp^2 + 2m = 2m\,(5m^2p - 3p^2 + 1)$.

5. Factorize the following :

(*i*) $mn + n$ (*ii*) $mx - m$

(*iii*) $5 - 15y$ (*iv*) $5mn - 5m$

(*v*) $-15ax - 20ay$ (*vi*) $-2mn - 4n$

6. (*i*) $a^2 - ab$ (*ii*) $x^4 - x^2$

(*iii*) $y^3 - y^4$ (*iv*) $p^5 - p^7 - p^9$

(*v*) $-b^5 + b^4 + 2b^3$ (*vi*) $-n^6 - n^8 - n^{12}$

(*vii*) $6z^4 - 12z^6$ (*viii*) $x^2y - xy^2$

(*ix*) $6a^2x + 12ax^2$ (*x*) $18\,ab^3 - 9\,b^4$

7. (*i*) $a^m + a^{m+1}$ (*ii*) $a^{3n} - a^{2n}$

(*iii*) $5^n - 5^{n-2}$ (*iv*) $c^{2\lambda+1} - c^{2\lambda} - 2c^{2\lambda-1}$

Multiplier may also be a polynomial. For example in th polynomial,

$$5\,(a-b) - 4x\,(a-b) + 7y\,(a-b),$$

we can see that $(a - b)$ is the common multiplier.
We shall obtain

$$5\,(a-b) - 4x\,(a-b) + 7y\,(a-b) = (a-b)(5 - 4x + 7y)$$

Factorize :

8. (*i*) $3a\,(a-b) + 2b\,(a-b)$

(*ii*) $7q\,(p-q) - 2p\,(p-q)$

(*iii*) $2b\,(x+y) - c\,(x+y) - 2\,(x+y)$

9. (*i*) $2m\,(x-3) - 5n\,(3-x)$

(*ii*) $m^2\,(a-2) + m\,(2-a)$

(*iii*) $2x\,(a-b) - 3y\,(b-a)$

(*iv*) $2a\,(x-y) - (y-x)$

10. Prove the following identities :

(*i*) $(a-2b)\,(a^2 - 5ab + b^2) - (a-2b)\,(a^2 - 6ab + b^2)$
$= ab\,(a-2b)$

(*ii*) $(a-3c)\,(2a^2 - 7ac - c^2) + (3c-a)\,(a^2 - 7ac - c^2)$
$= a^2\,(a-3c)$

11. Factorize and find the numerical values of the following polynomials :

(*i*) $7ax^2 - 14a^2x$ if $x = -4$, $a = -0{\cdot}5$

(*ii*) $a(b - c) + 3(c - b) + (b - c)$ if $a = 2$, $b = 1{\cdot}007$ and $c = -0{\cdot}06$.

12. Solve the following equations :

(*i*) $x^2 - 4x = 0$ (*ii*) $7x^2 + 2x = 0$

13. For papering or whitewashing the walls of a room it is necessary to know the area of the four walls. The area is given by the following formula :

$$A = 2h(l + b);$$

where l denotes the length, b denotes the breadth, and h the height of the room. For obtaining this formula look at the figure given below :

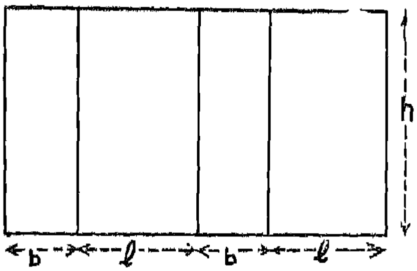

Fig. 4·1

Measure the length, breadth and height of your class-room and calculate the area of its walls. Work out the expenses of whitewashing or papering the walls.

§ 2. Method of Grouping

Factorize the following expression :

$$c(a - b) + d(a - b)$$

Taking the common multiplier out we get :

$$c\,(a - b) + d\,(a - b) = (a - b)(c + d)$$

Had the expression been as $ac - ad - bc + bd$ we should not have been able to find a common multiplier of this polynomial

However we can see that the first two terms have the common multiplier c and the remaining terms have the common multiplier d. So we can put these terms into two groups, one group having c as the common multiplier and the other having d as common multiplier. We can write the polynomial as :

$$ac - bc + ad - bd = (ac - bc) + (ad - bd).$$

Taking out the common multiplier from each group we have :

$$\begin{aligned} ac - bc + ad - bd &= (ac - bc) + (ad - bd) \\ &= c\,(a - b) + d\,(a - b). \end{aligned}$$

Now we see that $a - b$ is the common multiplier in each of the terms. On taking it out we shall obtain

$$\begin{aligned} ac - bc + ad - bd &= c\,(a - b) + d\,(a - b) \\ &= (a - b)(c + d). \end{aligned}$$

Example 1

Factorize the polynomial $2ac + 3b - bc - 5a$.

If we group the terms of this polynomial as $(2ac + 3b) -$ we do $(bc+6a)$, not get the common multiplier for any group.

Let us try to group it in some other way :

$$2ac + 3b - bc - 6a = (2ac - bc) + (3b - 6a).$$

On taking out the common multiplier we shall obtain

$$\begin{aligned} 2ac + 3b - bc - 6a &= (2ac - bc) + (3b - 6a) \\ &= c\,(2a - b) + 3\,(b - 2a) \\ &= c\,(2a - b) - 3\,(2a - b) \\ &= (2a - b)(c - 3). \end{aligned}$$

We could also group the polynomials as :

$$2ac + 3b \quad bc - 6a \quad 2ac - 6a + 3b - bc$$

The factors will be the same as shown above.

14. Put the last two terms into the brackets and factorize :

(*i*) $2a(x + y) + x + y$ (*ii*) $k(p - q) + p - q$

(*iii*) $3b(a + b) \quad a \quad b$ (*iv*) $2y(x - y) - x + y$

15 Factorize :

(*i*) $a(x - y) + bx - by$ (*ii*) $ax - ay + bx - by$

(*iii*) $m^2 + mn - 5m \quad 5n$ (*iv*) $x^2 - xy - 2x + 2y$

16. Calculate the values of the following expressions by the easiest method :

(*i*) $43 \quad 29 + 27 \quad 11 + 43 \times 11 + 27 \times 29$

(*ii*) $12{\cdot}7 \times 3{\cdot}4 + 8{\cdot}2 \times 1{\cdot}4 - 1{\cdot}4 \times 12{\cdot}7 - 3{\cdot}4 \times 8\,2$

17. Factorize :

(*i*) $4x^2 - 4xz - 3x + 3z$

(*ii*) $3ax - 4by - 4ay + 3bx$

(*iii*) $10a^2 + 21xy - 14ax - 15ay$

18. Factorize and calculate the values :

(*i*) $a^2b \quad b + ab^2 - a$, if $a = -1\frac{5}{8}, b = \frac{-8}{13}$

(*ii*) $3x^4 \quad 2y^4 - 6x^2y^2 + xy$, if $x = \frac{2}{3}, y = \frac{1}{2}$

19. Solve the equation :

$$x^3 + 3x^2 - 4x - 12 \quad 0$$

Hint : First factorize the expression

20. Prove the following identity :

$$x^2 - xy + 2x - 2y = (x - y)(x + 2)$$

21. Factorize :

(*i*) $ax^2 + bx^2 - bx - ax + a + b$

(*ii*) $ax^2 + bx^2 - bx - ax + cx^2 - cx$

§ 3. Factorization of the Polynomials with the Help of Formulas

If we write down the multiplication formulas in inverse order we shall obtain :

(1) $a^2 - b^2 = (a + b)(a - b)$

(2) $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

(3) $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

(4) $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$

$$= (a + b)(a + b)(a + b)$$

(5) $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$

$$= (a - b)(a - b)(a - b)$$

(6) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(7) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Let us take some examples.

Example 2

$$25b^2 - 81a^2 = (5b)^2 - (9a)^2.$$

We have the difference of the squares of the terms $5b$ and $9a$.

We can factorize the given expression in the following way with the help of the first formula :

$$\begin{aligned} 25b^2 - 81a^2 &= (5b)^2 - (9a)^2 \\ &= (5b + 9a)(5b - 9a) \end{aligned}$$

Example 3

$$x^2 - 10x + 25 = x^2 - 2(x \times 5) + 5^2$$

In this case we can use the third formula :

$$\begin{aligned} x^2 - 10x + 25 &= x^2 - 2\,(x \times 5) + 5^2 \\ &= (x - 5)^2 \\ &= (x - 5)(x - 5) \end{aligned}$$

Example 4

$$a^3 + 18a^2 + 108a + 216 = a^3 + 3\,(a^2 \times 6) + 3\,(a \times 6^2) + 6^3$$

If we use the 4th formula, we shall obtain

$$\begin{aligned} a^3 + 18a^2 + 108a + 216 &= a^3 + 3\,(a^2 \times 6) + 3\,(a \times 6^2) + 6^3 \\ &= (a + 6)^3 \\ &= (a + 6)(a + 6)(a + 6) \end{aligned}$$

22. Factorize (orally) :

(*i*) $x^2 - y^2$ (*ii*) $m^2 - n^2$ (*iii*) $a^2 - 4$

(*iv*) $t^2 - 9$

23. Find orally the values of :

(*i*) $85^2 - 15^2$ (*ii*) $388^2 - 312^2$

(*iii*) $\left(5\frac{3}{4}\right)^2 - \left(2\frac{1}{4}\right)^2$ (*iv*) $(5{\cdot}6)^2 - (4{\cdot}4)^2$

24. Factorize :

(*i*) $c^2 - 36$ (*ii*) $1 - m^2$

(*iii*) $m^2 - 4n^2$ (*iv*) $36q^2 - 25$

25. (*i*) $a^2b^2 - 4$ (*ii*) $49 - p^2q^2$

(*iii*) $\frac{4}{9}x^2 - \frac{16}{25}y^2$ (*iv*) $1 - 0{\cdot}01a^2$

26. $100a^4 - 81b^6$ (*ii*) $a^6 - 4$

27. In figure 4.2 you see the plan of a plot. In the centre of the plot there is a building.

(*i*) Work out a formula for calculating the area of the plot (*a*) occupied by the building, (*b*) not occupied by the building.

(*ii*) What will be the two areas if

$a = 45{\cdot}5\ m$ and $b = 14{\cdot}5\ m$?

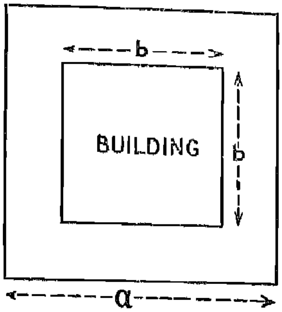

Fig 4.2

Factorize :

28. (*i*) $(a + b)^2 - c^2$ (*ii*) $(p - q)^2 - r^2$

(*iii*) $(3a - b)^2 - 9b^2$ (*iv*) $(3a + 1)^2 - \frac{1}{4} a^2$

29. (*i*) $p^2 - (q - r)^2$ (*ii*) $(b - c)^2 - 0{\cdot}04b^2$
(*iii*) $9a^2 - (b^2 + c^2)^2$ (*iv*) $1 - (2a - 3b)^2$
(*v*) $4\,(x - y)^2 - 9$ (*vi*) $16\,(a - 5b)^2 - 25b^2$

30. (*i*) $(m + n)^2 - (m - n)^2$ (*ii*) $(x + 2y)^2 - (x - 2y)^2$
(*iii*) $(m + n + p)^2 - (m - n - p)^2$
(*iv*) $(a + b - c)^2 - (a - b + c)^2$

31. Solve the following equations :

(*i*) $x^2 - \frac{1}{4} = 0$ (*ii*) $x^2 - 1{\cdot}44 = 0$

(*iii*) $9x^2 = 16$

32. Use the formulas $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and factorize the following polynomials :

(*i*) $x^2 - 2x + 1$ (*ii*) $4a^2 + 4a + 1$
(*iii*) $9m^2 - 6m + 1$ (*iv*) $-a^2 - 2a - 1$
(*v*) $-6a - a^2 - 9$ (*vi*) $x^4 - 2bx^2 + b^2$
(*vii*) $-x^4 - 2nx^2 - n^2$ (*viii*) $9c^2 + 12cd^2 + 4d^4$
(*ix*) $25x^4 - 10x^2y^2 + y^4$ (*x*) $36p^4 + 12p^2p^2 + q^4$

33 Supply the missing terms in the following :

(*i*) $64a^2 - \ldots\ldots + 49b^2 = (\ldots\ldots - \ldots\ldots)^2$

(*ii*) $144x^2 + \ldots\ldots + 25y^2 = (\ldots\ldots + \ldots\ldots)^2$

(*iii*) $36a^6b^4 + 156a^4b^2c^4 + \ldots\ldots = (\ldots\ldots + \ldots\ldots)^2$

34. Use the formulas $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$ and factorize ;

(*i*) $a^3 + 3a^2b + 3ab^2 + b^3$ (*ii*) $x^3 - 3x^2y + 3xy^2 - y^3$

(*iii*) $m^3 + 6m^2n + 12mn^2 + 8n^3$

(*iv*) $a^3 - 6a^2b + 12ab^2 - 8b^3$

(*v*) $64a^6 - 48a^4b + 12a^2b^2 - b^3$

(*vi*) $x^9 - 15x^6y + 75x^3y^2 - 125y^3$

(*vii*) $\frac{1}{8}k^3 + \frac{3}{4}k^2p^2 + 1\frac{1}{2}kp^4 + p^6$

(*viii*) $b^{15} - b^{10}c^2 + \frac{1}{3}b^5c^4 - \frac{1}{27}c^6$

35. Supply the missing terms in the following :

(*i*) $125a^3 - \ldots\ldots + \ldots\ldots - 8b^3 = (\ldots\ldots - \ldots\ldots)^3$

(*ii*) $64x^3 + \ldots\ldots + \ldots\ldots + 27y^6 = (\ldots\ldots + \ldots\ldots)^3$

(*iii*) $27a^9 - 108a^6b^2 + \ldots\ldots - \ldots\ldots = (\ldots\ldots - \ldots\ldots)^3$

36. Factorize :

(*i*) $8d^3 - p^3$

(*ii*) $64a^3 + b^3$

(*iii*) $64k^3 - 1$

(*iv*) $216x^3 + 1$

(*v*) $8p^3 + 27q^3$

(*vi*) $125x^3 - 64z^3$

(*vii*) $\frac{1}{27}a^3 - 1000$

(*viii*) $\frac{8}{125}b^3 + \frac{1}{27}$

(*ix*) $m^3 - 27$

(*x*) $1 - q^3$

37. Factorize and find the numerical values of :

(*i*) $2x^2 - 2y^2$ if $x = 55, y = 45$; *and* $x = 86, y = 36$

(*ii*) $\frac{1}{2}a^2 + ab + \frac{1}{2}b^2$ if $a = 64, b = 36$

38. Solve the equations :

(*a*) $121x^2 + 110x + 25 = 0$

(*b*) $y^3 - 30y^2 + 300y - 1000 = 0$

39. Factorize :

(*i*) $x^3 - x$ (*ii*) $5a^3 - 5a$

(*iii*) $p^4q^2 - p^2q^4$ (*iv*) $5a^3 + 10ab + 5b^2$

(*v*) $2a - 4ab + 2ab^2$ (*vi*) $m^2 + 2mn + n^2 - p^2$

(*vii*) $x^2 + 2xy + y^2 - 1$ (*viii*) $9 - x^2 + 2xy - y^2$

(*ix*) $4 - a^2 - 2ab - b^2$ (*x*) $a^2 - b^2 - a + b$

(*xi*) $a^2 + 2ab + b^2 - c^2 - 2cd - d^2$

40. Factorize :

(*i*) $x^5 - x^3 + x^2 - 1$ (*ii*) $a^4 + a^3 + a + 1$

(*iii*) $x^4 + x^3 - x - 1$ (*iv*) $x^3 - x^2y - xy^2 + y^3$

(*v*) $m^4 - n^4$ (*vi*) $(a + b)^3 - (a - b)^3$

(*vi*) $x^4 - x^3 - x + 1$ (*viii*) $c^5 - c^4 - c + 1$

We can factorize a quadratic trinomials in two ways :

(1) by completing the square, for example,

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 6x + 9 - 9 + 8 \\ &= (x + 3)^2 - 1^2 \\ &= (x + 3 + 1)(x + 3 - 1) \\ &= (x + 4)(x + 2). \end{aligned}$$

(2) by breaking the middle term into the sum of two terms, for example,

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + 2x + 4x + 8 \\ &= (x^2 + 2x) + (4x + 8) \\ & x(x + 2) + 4(x + 2) \\ & (x + 2)(x + 4) \end{aligned}$$

41. Factorize the following trinomials :

(*i*) $x^2 - 5x + 6$ (*ii*) $x^2 - x - 12$

(*iii*) $x^2 + x - 12$ (*iv*) $2a^2 + 10a + 12$

(*v*) $3p^2 + 27p + 54$

We know from arithmetic that

$$\frac{2772}{28} = \frac{2^2 \times 7 \times 3^2 \times 11}{2^2 \times 7}$$

$$= 3^2 \times 11 = 99$$

Similarly we can solve the division of polynomials in algebra.

Example 5

Divide $x^5 - a^4x$ by $x^2 + ax$

$$(x^5 - a^4x) \div (x^2 + ax) = \frac{x^5 - a^4x}{x^2 + ax}$$

$$= \frac{x(x^4 - a^4)}{x(x + a)} = \frac{x(x^2 + a^2)(x^2 - a^2)}{x(x + a)}$$

$$= \frac{x(x^2 + a^2)(x + a)(x - a)}{x(x + a)}$$

$$= (x^2 + a^2)(x - a)$$

Having known the multiplication formulas, we can write the following formulas of division ;

$$\frac{a^2 - b^2}{a - b} = a + b$$

$$\frac{a^2 - b^2}{a + b} = a - b$$

$$\frac{a^3 - b^3}{a - b} = a^2 + ab + b^2$$

$$\frac{a^3 + b^3}{a + b} = a^2 - ab + b^2$$

42. Divide (orally) :

(*i*) $x^2 - 9$ by $3 + x$ (*ii*) $a^2 - 4$ by $2 - a$

(*iii*) $m^2 - 1$ by $1 + m$ (*iv*) $k^2 - 4$ by $2 - k$

43. Divide :

(*i*) $16 - m^2n^2$ by $mn + 4$

(*ii*) $\frac{1}{4}a^2 - \frac{1}{9}b^2$ by $\frac{1}{2}a + \frac{1}{3}b$

(*iii*) $\frac{4}{25}x^2 - \frac{9}{16}y^2$ by $\frac{3}{4}y - \frac{2}{5}x$

(*iv*) $\frac{9}{16}c^2 - \frac{4}{25}d^2$ by $\frac{2}{5}d - \frac{3}{4}c$

44. Simplify :

(*i*) $(x^2 - 4x + 4) \div (x - 2)$

(*ii*) $(m^3 + 3m^2n + 3mn^2 + n^3) \div (m^2 + 2mn + n^2)$

(*iii*) $(p^3 - 3p^2q + 3pq^2 - q^3) \div (p^2 - 2pq + q^2)$

45. Simplify :

(*i*) $\frac{8a^3 - 27b^3}{2a - 3b}$ (*ii*) $\frac{27x^6 + 8y^3}{3x^2 + 2y}$

(*iii*) $\frac{27b^6 + 1}{3b^2 + 1}$ (*iv*) $\frac{8b^6 - 1}{1 - 2b^2}$

CHAPTER V

Fractions

§ 1. Notion of Algebraic Fractions

It has been said earlier, that if division of algebraic expression is not possible, we can express the quotient as a fraction, taking dividend as the numerator and the divisor as the denominator.

Examples of fractional expressions are :

$$\frac{m}{2a}, \quad \frac{a^2-x}{2b-y}, \quad \frac{3(p+q)-11}{7x^4}, \quad \frac{(a+b)^3-(x-y)^2}{m^2-p+n^2}$$

The numerator and denominator can be fractions also. For example,

$$\frac{\frac{a+x}{y}}{4a^3}; \qquad \frac{\frac{a-b}{a+b}}{\frac{4}{3x}}$$

Such algebraic expressions are called *Algebraic Fractions*.

Definition : An algebraic expression represents an algebraic fraction, if the numerator and the denominator of the expression are polynomials. Arithmetical fraction is a particular case of algebraic fraction.

We can write any polynomial as a fraction, for example,

$$a^2 - y = \frac{a^2 - y}{1}$$

Letters of the numerator can be assigned any values (if there is no limitation).

Permissible values of the letters of the denominator are such values as do not make it zero.

1. Write the quotients in each of the following division-sums of the algebraic expressions, as fractions :

(*a*) $a \div b$ (*b*) $5 \div x$

(*c*) $y \div z$ (*d*) $(a + b) \div 4$

(*e*) $9 \div (m - n)$ (*f*) $(x + y) \div (x - y)$

(*g*) $3x \div (2a + 3b)$ (*h*) $(a^2 - b) \div (2x^2 + y - 1)$

2. A typist can type some written material in a hours ; another can type the same in b hours. What part of the material can be typed by each typist in one hour ? What part can be typed by both together in one hour ? Calculate, in how many hours they will finish the work together if :

(*i*) $a = 4, b = 6,$ (*ii*) $a = 2\frac{1}{2}, b = 1\frac{1}{4}$

3. The number of persons in a town is a. The population of this town increases by p per cent each year. How many persons will there be after one year ?
What will be the result if :

(*i*) $a = 15{,}000, p = 5,$ (*ii*) $a = 70{,}000, p = 3{\cdot}4,$

(*iii*) $a = 1{,}000{,}000, p = 8{\cdot}5.$

4· A scooter went with a speed of v km/hr for t hours and with a speed of v_1 km/hr for t_1 hours. Find the average speed, say

v_0 km/hr of the scooter. What will be the value of v_0 if $t = 2{\cdot}2$ hrs, $v = 55$ km/hr, $t_1 = 1{\cdot}5$ hrs, $v_1 = 7{\cdot}2$ km/hr ?

5. A tourist sailed a km with the current for t hours. Find how much time will he take to come back if the speed of the current is v km/hr ? If $a = 18{\cdot}2$ km, $t = 2$ hrs, 12 mts and $v = 1{\cdot}8$ km. What will be the time nearest to the minute taken by tourist in coming back ?

If we substitute in a fraction the given numerical values of the letters, we shall obtain the numerical values of the fraction. For example, if we take $a = 6$, $b = 3$; then 2 is the numerical value of the fraction $\frac{a}{b}$.

Again, if we take $a = 0$, $b = 5$, then the numerical value of the fraction $\frac{a}{b}$ is equal to 0.

However, a fraction which has, zero as its denominator is meaningless.

6. What values do the following fractions have :

(*i*) $\frac{0}{a}$ $(a \neq 0)$ (*ii*) $\frac{0}{2b}$ $(b \neq 0)$ (*iii*) $\frac{0}{x - y}$ $(x \neq y)$

7. What values of m will make the following fractions equal to zero :

(*i*) $\frac{m - 2}{3}$ (*ii*) $\frac{m + 5}{m}$

(*iii*) $\frac{m - 3}{m + 1}$ (*iv*) $\frac{m\,(m - 10)}{m + 15}$

(*v*) $\frac{(m + 2)(m - 3)}{m + 5}$ (*vi*) $\frac{(m + 1)(m - 4)}{m - 3}$

8. For what value of x the following fractions will be meaningless :

(*i*) $\frac{5}{x - 1}$ (*ii*) $\frac{1}{x + 1}$

(*iii*) $\frac{x}{x - c}$ (*iv*) $\frac{x}{2x - 8}$

(*v*) $\frac{x - 1}{x + 1}$

9. Fill the following table (orally) :

x	-3	-2	-1	0	1	2	3	9	15
$z = \frac{x - 3}{x + 1}$									

10. (*a*) Write down algebraic fractions which will be meaningless if :

(*i*) $x = 8$ (*ii*) $x = 7$

(*iii*) $x = 4$ and $x = -4$ (*iv*) $x = 5$

(*v*) $x = 0$

(*b*) Write down algebraic fractions which will be equal to zero when :

(*i*) $a = 0$ (*ii*) $a = 10$

(*iii*) $a = -4$ (*iv*) $a = 3$

(*v*) $a = 20$

11. Solve the following equations and determine for what values of a the equations have no solutions :

(*i*) $ax - 6 = x$ (*ii*) $ax + 7 = 11 - 2x$

(*iii*) $ax - a = 10x + 9$ (*iv*) $a - ax = 2 - 3x$

§ 2. Basic Property of an Algebraic Fraction

The fraction, the numerator and the denominator of which are any rational numbers (the denominator is not equal to zero), has the following property :

The value of a fraction is not changed if the numerator and the denominator are multiplied by the same number not equal to zero. For example,

$$\frac{a}{b} = \frac{am}{bm} \text{ where } m \neq 0$$

We can similarly formulate the basic property for algebraic fractions.

If the numerator and the denominator of an algebraic fraction are multiplied by the same polynomial, we shall obtain the fraction equal to the given fraction.

Let us explain it through an example. Multiply the numerator and the denominator of the fraction $\frac{x+2}{x-3}$ by a binomial $x-2$.

We get the fraction $\frac{(x+2)(x-2)}{(x-3)(x-2)}$

Take any value x except 3 and 2 (the denominator of the fraction will be zero if $x = 3$ or 2).

Put this value of x in the two fractions $\frac{(x+2)}{(x-3)}$ and $\frac{(x+2)(x-2)}{(x-3)(x-2)}$. The result obtained is the same.

So, $$\frac{x+2}{x-3} = \frac{(x+2)(x-2)}{(x-3)(x-2)}$$

or $$\frac{x+2}{x-3} = \frac{x^2-4}{x^2-5x+6}$$

12. Which of the following equations are identities :

(*i*) $\frac{a}{a+b} = \frac{an}{an+bn}$ (*ii*) $\frac{am+b}{cn+d} = \frac{a+b}{c+d}$

(*iii*) $\frac{x+1}{y+3} = \frac{x^2+1}{xy+3}$ (*iv*) $\frac{b}{c} = \frac{by-d}{c-d}$

13. Will the values of the following fractions be changed if x and y are simultaneously multiplied by 10 ? If so, point out the cases from the following :

(*i*) $\frac{x}{y}$ (*ii*) $\frac{x}{3y}$ (*iii*) $\frac{x+y}{x-y}$ (*iv*) $\frac{x-1}{y+1}$

(v) $\frac{3x}{x + y}$ (vi) $\frac{x^2}{x + y}$ (vii) $\frac{y - 4}{x - 4}$

§ 3. Reduction of Fractions

We can write the previous equality in the inverse order, viz :

$$\frac{x^2 - 4}{x^2 - 5x + 6} = \frac{x + 2}{x - 3}$$

It shows how we have transformed the fraction $\frac{x^2 - 4}{x^2 - 5x + 6}$ into a simpler fraction $\frac{x + 2}{x - 3}$, by dividing the numerator and the denominator by a common factor $x - 2$ If the numerator and the denominator of the fraction are divided by the same polynomial, the value of the fraction is not changed. We can use this property for simplifying a fraction by dividing both its numerator and denominator by the same common factor. Such simplification is called **'Reduction of a Fraction to lowest terms'**.

14. Reduce the following fractions to their lowest terms :

(*i*) $\frac{8}{12}$, $\frac{45}{120}$, $\frac{84}{210}$, $\frac{435}{1215}$

(*ii*) $\frac{15a}{20b}$, $\frac{ab}{ac}$ (*iii*) $\frac{10mn}{15mp}$

(*iv*) $\frac{8ax}{16ay}$ (*v*) $\frac{2a^2}{3ab}$

15. (*i*) $\frac{5x}{5xy}$ (*ii*) $\frac{24xy}{56x^4y}$

(*iii*) $\frac{45a^8}{63a^4b}$ (*iv*) $\frac{270a^{10}b^8c^7}{405a^8b^{12}c^{14}}$

(*v*) $\frac{1080a^{11}b^7c^{15}}{1008a^{10}b^{14}c^{16}}$

16. (*i*) $\frac{3a(x+y)^2}{9a^2(x+y)}$ (*ii*) $\frac{10a^2b(x-y)^2}{15a^4b(x-y)^3}$

(*iii*) $\frac{3(a-b)(a-c)^2}{6(a-b)(a-c)}$ (*iv*) $\frac{a(b+c)}{a(b+c)}$

17. (*i*) $\frac{a^2}{a^k}$ (*ii*) $\frac{b^3}{b^n}$ (*iii*) $\frac{x^3}{x^{n+3}}$

(*iv*) $\frac{z}{z^{k-3}}$ (*v*) $\frac{4x^{n+3}}{x^{n+3}}$

According to the basic property of a fraction, the value of the fraction is not changed if its numerator and denominator both are multiplied by -1. For example :

$$\frac{7}{12} = \frac{7 \times (-1)}{12 \times (\ 1)} = \frac{-7}{-12}$$

and

$$\frac{2}{-3} = \frac{2 \times (-1)}{-3 \times (\ 1)} = \frac{-2}{3}$$

Multiplication of any number by -1 changes the sign of this number from plus to minus and from minus to plus. And so the value of a fraction is not changed if we change the sign of both the numerator and the denominator by the opposite signs. Thus

$$\frac{a}{b} = \frac{-a}{-b}$$

If we change the sign of the numerator or of the denominator only, we shall obtain the fraction opposite to the given fraction. For example,

$$\frac{2}{3} \text{ and } \frac{-2}{3}\,;\ \frac{2}{3} \text{ and } \frac{2}{-3}$$

$$-\left(\frac{2}{3}\right) = \frac{-2}{3} \text{ and } -\left(\frac{2}{3}\right) = \frac{2}{-3}$$

$$\frac{a}{b} = -\frac{-a}{b} \text{ and } \frac{a}{-b} = -\frac{a}{b}\,;$$

$$\frac{a}{b} = \frac{-a}{-b} \text{ and } \frac{a}{b} = -\frac{a}{-b}$$

If we change the sign of one of the terms of the fraction and also the sign of the fraction by the opposite sign, the value of the fraction is not changed. For example,

$$\frac{a}{b} = -\frac{-a}{b} = -\frac{a}{-b}$$

18. Without changing the value of a fraction, remove the sign from the numerator and the denominator of the fractions given below :

(*i*) $\frac{-2a}{-5b}$ (*ii*) $\frac{8c^2}{-15x}$

(*iii*) $-\frac{-3m}{4n}$ (*iv*) $-\frac{-x}{-y}$

(*v*) $-\frac{3x^2y}{-10z}$ (*vi*) $\frac{-4ab}{-7cd}$

19. Without changing the values of the fractions, transform the following fractions in such a way that they have minus sign before them :

(*i*) $\frac{m}{1-m}$ (*ii*) $\frac{a-b}{c+d}$

(*iii*) $\frac{a-x}{b-x}$ (*iv*) $\frac{-a-b}{c+d}$

20. Reduce the following fractions to their lowest terms :

(*i*) $\frac{a(x-a)}{b(a-x)}$ (*ii*) $\frac{5a(x-y)}{15a(y-x)}$

(*iii*) $\frac{3m(x-1)}{9m^2(1-x)}$

21. (*i*) $\frac{ac-bc}{ac+bc}$ (*ii*) $\frac{ax+bx}{ax-bx}$

(*iii*) $\dfrac{a^2}{a^2 + ab}$ (*iv*) $\dfrac{xy}{x - xy}$

(*v*) $\dfrac{ac - bc}{c^2 + cd}$ (*vi*) $\dfrac{k^2 + k}{kx - ky}$

22. (*i*) $\dfrac{x^2 - 4x + 4}{x^2 - 4}$ (*ii*) $\dfrac{a^3 - b^3}{2a - 2b}$

(*iii*) $\dfrac{1 - x^3}{3 + 3x + 3x^2}$ (*iv*) $\dfrac{x - x^2}{x^2 - 1}$

(*v*) $\dfrac{a^2 - 1}{1 - a}$ (*vi*) $\dfrac{m - n}{(n - m)^2}$

23. (*i*) $\dfrac{x^2 + 2xy + y^2}{x^2 - y^2}$ (*ii*) $\dfrac{3a^2 - 6ab + 3b^2}{6a^2 - 6b^2}$

(*iii*) $\dfrac{a^3 + b^3}{a^2 - b^2}$ (*iv*) $\dfrac{3m^2 - 3n^2}{6m^3 + 6n^3}$

(*v*) $\dfrac{x^4 - y^4}{x^3 - y^3}$ (*vi*) $\dfrac{a^2 + ab + b^2}{a^3 - b^3}$

24. (*i*) $\dfrac{ax + ay - bx - by}{ax - ay - bx + by}$ (*ii*) $\dfrac{(a + b)^2 - c^2}{a + b + c}$

(*iii*) $\dfrac{a^2 + b^2 - c^2 + 2ab}{a^2 - b^2 + c^2 + 2ac}$ (*iv*) $\dfrac{1 - 3y + 3y^2 - y^3}{z - zy + x - xy}$

25. (*i*) $\dfrac{a^2 - b^2}{a^2 - a - b - b^2}$ (*ii*) $\dfrac{a^3 + 1}{6a^2 + 12a + 6}$

(*iii*) $\dfrac{3x^2y - xy^2}{3x^3 - 3xy^2 - x^2y + y^3}$ (*iv*) $\dfrac{a^2b + ab^2}{a^3 + b^3 + 3ab\,(a + b)}$

26. (*i*) $\dfrac{a}{a}$ (*ii*) $\dfrac{-b}{|b|}$

(*iii*) $\dfrac{x - y}{x - y}$ (*iv*) $\dfrac{|m - n|}{m^2 - mn}$

27. Solve for x the following equations :

(*i*) $2x = 3d$ (*ii*) $ax = b + c$

(*iii*) $a^2x - ab = a$ (*iv*) $4mn - 2nx = 6n^2$

(*v*) $bx - cx = a$ (*vi*) $ax + x = a^2 + 2a + 1$

(*vii*) $m^2x + 2mnx + n^2x = 3m^2 - 3n^2$

§ 4. Reducing Fractions to the Common Denominator

The basic property of a fraction helps us to transform an algebraic fractions with different denominators into fractions with the same denominators.

Such a transformation of fractions is called 'reducing the fractions to the common denominator'.

Let us illustrate it through examples.

(*a*) Fractions with Monomial Denominators

Example 1

$$\frac{n}{4ab^5}\,;\ \frac{n}{6a^3c^2}\,;\ \frac{p}{9a^4b^2}$$

Let us work out the common denominator of these fractions.

1. The number 36 is the least common multiple for the coefficients 4, 6 and 9.
2. The common denominator must contain the factor a^4.
3. The common denominator must contain the factor b^5.
4. The common denominator must contain the factor c^2.

So, we shall obtain the monomial $36a^4\,b^5\,c^3$ as the common denominator for the given fractions. In order to obtain the fractions with denominator $36a^4\,b^5\,c^2$, it is necessary to multiply both terms of each fraction by $9a^3c^2$, $6ab^5$ and $4b^3c^2$ respectively, so that

$$\frac{m}{4ab^5} = \frac{9a^3\,c^2m}{36a^4b^5c^2}\,;\ \frac{n}{6a^3c^2} = \frac{6ab^5n}{36a^4b^5c^2}\,;$$

$$\frac{p}{9a^4b^2} = \frac{4b^3c^2p}{36a^4b^5c^2}$$

Conclusion : **The simplest common denominator of the fractions with monomial denominators is the least common multiple of the coefficients of the denominators multiplied by all different letters in the denominators and each letter is taken with the highest exponent occurring in any of the denominators.**

28. Reduce the following fractions to the common denominator :

(*a*) $\frac{1}{24a^5x^7y}$ and $\frac{7}{60a^6x^4}$ (*b*) $\frac{xy}{18a^{10}b^5}$ and $\frac{yz}{45a^6b^5c^3}$

(*c*) $\frac{a}{x^n}$ and $\frac{b}{x^{n+2}}$

(*b*) Fractions with Polynomial Denominators

Example 2

Consider the following fractions ;

$$\frac{a}{4x^2-4y^2}\,;\quad \frac{b}{5x^2+10xy+5y^2}\,;\quad \frac{c}{10x^2-20xy+10y^2}$$

Let us first factorize the denominators.

$$4x^2-4y^2=4(x^2-y^2)=4(x+y)(x-y)$$

$$5x^2+10xy+5y^2=5(x^2+2xy+y^2)=5(x+y)^2$$

$$10x^2-20xy+10y^2=10(x^2-2xy+y^2)=10(x-y)^2$$

Let us work out the common denominator in the same way as for monomial denominators. It will be

$$20(x+y)^2(x-y)^2$$

Now change each fraction to this denominator. We shall obtain the following fractions :

$$\frac{a}{4x^2-4y^2}=\frac{a}{4(x+y)(x-y)}$$

$$=\frac{5a(x+y)(x-y)}{20(x+y)^2(x-y)^2}$$

$$\frac{b}{5x^2 + 10xy + 5y^2} = \frac{b}{5(x+y)^2}$$

$$= \frac{4b(x-y)^2}{20(x+y)^2(x-y)^2}$$

$$\frac{c}{10x^2 - 20xy + 10y^2} = \frac{c}{10(x-y)^2}$$

$$= \frac{2c(x+y)^2}{20(x+y)^2(x-y)^2}$$

Conclusion : In order to reduce fractions with polynomial denominators to a common denominator, we have

(*i*) to factorize the denominators

and (*ii*) to find the least common multiple of the denominators.

This least common multiple will be the common denominator for all algebraic fractions.

29. Reduce the following fractions to a common denominator :

(*a*) $\frac{m}{(a-b)^2}$ and $\frac{p}{a^2 - b^2}$

(*b*) $\frac{f}{abc - a^2d}$ and $\frac{t}{b^2c - abd}$

(*c*) $\frac{5}{4b^2 - 1}$, $\frac{6s}{6b - 3}$ and $\frac{3p}{4b + 2}$

§ 5. Operations with Fractions

(*a*) Addition of Fractions

30. Add (orally) :

$\frac{3}{7}$ and $\frac{2}{7}$, $\frac{a}{4}$ and $\frac{b}{4}$, $\frac{5a}{6}$ and $\frac{a}{6}$, $\frac{2p^2}{5}$ and $\frac{p^2}{5}$

Example 3

A school was given a copy-books for class I and b copy-books for class II. If each pupil got m copy-books, find the number of pupils in the both classes ?

Solution

I Method

(1) How many pupils are there in class I ?

$$a \div m = \frac{a}{m} \text{ pupils}$$

(2) How many pupils are there in class II ?

$$b \div m = \frac{b}{m} \text{ pupils}$$

(3) The number of pupils in both the classes

$$= \frac{a}{m} + \frac{b}{m}$$

II Method

(1) How many copy-books are given to both the classes ?

$a + b$ copy-books

(2) How many pupils are there in both the classes ?

$$\frac{a+b}{m}$$

After this we can compare the two results, namely

$$\frac{a}{m} + \frac{b}{m} \text{ and } \frac{a+b}{m}.$$

These results are equal, so

$$\frac{a}{m} + \frac{b}{m} = \frac{a+b}{m}$$

This equality is true for all values of a, b and m, except when $m = 0$.

We can express it in the other way also, *i.e.*,

$$\frac{a+b}{m} = \frac{a}{m} + \frac{b}{m}$$

If we read this equality in the inverse order, we shall obtain the equality :

$$\frac{a}{m} + \frac{b}{m} = \frac{a+b}{m}$$

Conclusion : In order to add the fractions having the same denominators, it is sufficient to add their numerators and then divide the sum thus obtained by the common denominator.

31. Find the sum of :

(1) $\frac{a+b}{3}$ and $\frac{a}{3}$ (2) $\frac{m-n}{a}$ and $\frac{m+n}{a}$

(3) $\frac{a+b}{x+a}$ and $\frac{a-b}{x+a}$ (4) $\frac{x+4}{a-2}$ and $\frac{x-3}{a-2}$

32. Simplify :

(1) $\frac{a}{x-1} + \frac{b}{1-x}$ (2) $\frac{m}{2p-q} + \frac{n}{q-2p}$

(3) $\frac{a+1}{a-1} + \frac{a-2}{1-a}$ (4) $\frac{m+n}{p-q} + \frac{m-4}{q-p}$

For the addition of fractions having different denominators, it is necessary to reduce them to the common denominator.

Example 4

Add the fractions $\frac{6}{6m}$ and $\frac{b}{8n}$.

It is necessary to change each of these fractions to the common denominator (24 *mn*).

Then $\frac{a}{6m} = \frac{4an}{24mn}$ and $\frac{b}{8n} = \frac{3bm}{24mn}$

$$\therefore \quad \frac{a}{6m} + \frac{b}{8n} = \frac{4an}{24mn} + \frac{3bm}{24mn} = \frac{4an+3bm}{24mn}$$

33. Find the sum :

(1) $\frac{3}{a}+\frac{5}{b}$ (2) $\frac{5x}{24}+\frac{7x}{30}$

(3) $\frac{x}{3}+\frac{2x}{10}+\frac{4x}{15}$

34. (1) $\frac{1}{4a}+\frac{1}{2b}$ (2) $\frac{3x-2}{5}+\frac{5x-3}{3}$

(3) $\frac{2a-3}{4}+\frac{5a+3}{3}$

35. (1) $\frac{x}{ab}+\frac{x}{ac}$ (2) $\frac{3c^2+5ab}{ac}+\frac{b^2-3ac}{bc}$

(3) $\frac{1}{m^4n^3}+\frac{2}{m^3n^4}$

§ 6. Subtraction of Fractions

Subtraction of fractions is an inverse operation with respect to the operation of addition. The rule of subtraction is given below :

For the two fractions having the same denominators, it is sufficient to subtract the numerator of the subtrahend from the numerator of minuend and to divide the difference thus obtained by the common denominator.

Example 5

Subtract $\frac{b}{m}$ from $\frac{a}{m}$:

$$\frac{a}{m}-\frac{b}{m}=\frac{a-b}{m}$$

36. Subtract :

(1) $\frac{a^2}{10}$ from $\frac{b}{10}$ (2) $\frac{x}{2}$ from $\frac{5x+1}{2}$

(3) $\frac{a+1}{4}$ from $\frac{a+3}{4}$ (4) $\frac{p-q}{m}$ from $\frac{3p-q}{m}$

37. Simplify :

(1) $\frac{2x}{a-b}-\frac{x}{b-a}$ (2) $\frac{5y^2}{a-2}-\frac{2y^2}{2-a}$

(3) $\frac{x+y}{x-y}-\frac{y+2x}{y-x}$ (4) $\frac{a}{x^2-1}-\frac{b}{1-x^2}$

For subtraction of the fractions with different denominators, it is necessary to reduce them to the common denominator and then to use the above mentioned rule.

38. Simplify :

(1) $\frac{15}{28}-\frac{13}{42}$ (2) $\frac{x}{5}-\frac{y}{8}$

(3) $\frac{13x}{120}-\frac{11x}{90}$ (4) $\frac{7}{x}-\frac{4}{y}$

39. Apply the operation of subtraction :

(1) $\frac{3}{a}-\frac{2}{ab}$ (2) $\frac{x}{ab}-\frac{y}{ac}$

(3) $\frac{4p+3a}{10}-\frac{2p-q}{15}$ (4) $\frac{2b^2-3a^2}{5}-\frac{5a^2-b^2}{4}$

40. Simplify :

(1) $\frac{x-1}{4}-\frac{x+3}{4}-\frac{x-3}{4}$

(2) $\frac{x+1}{a-b}-\frac{x+2}{b-a}-\frac{x-1}{a-b}$

(3) $\frac{5a}{6b^2c} - \frac{7b}{12ac^2} + \frac{11c}{18a^2b}$

(4) $\frac{5(2a-b)}{8} - \frac{3(a-4b)}{2} + \frac{7(a-b)}{6}$

(5) $\frac{x^3+xyz}{x^2y} + \frac{xyz+y^3}{y^2z} - \frac{xyz+z^3}{z^2x}$

(6) $\frac{3c-2b}{8bc} + \frac{a-7b}{12ab} + \frac{5a-c}{6ac} - \frac{2c-3b}{3bc} - \frac{3}{4a}$

41. (1) $m + \frac{1}{n}$ (2) $\frac{x^2+y}{x} - y$

(3) $5 - \frac{1}{b} - \frac{1}{a}$ (4) $a - b - \frac{a^2+b^2}{a}$

(5) $x - \frac{x-y}{8} + \frac{x+y}{4} - y$

(6) $\frac{2x-5y}{2} - y - \frac{3x-y}{4}$

42. Solve the following equations :

(1) $10 - \frac{3x-1}{2} = \frac{6x+3}{11}$

(2) $\frac{x+6}{2} + \frac{5-4x}{3} - \frac{7-x}{6} = 0$

(3) $\frac{7+9x}{4} - 1 + \frac{2-x}{9} = 7x$

(4) $x + 2\frac{1}{2} = \frac{4x+3}{4} - \frac{2-3x}{8}$

(5) $\frac{5z-6}{3} - \frac{5z+6}{12} = 1$

(6) $\frac{9p-5}{2} - \frac{3+2p}{3} - \frac{8p-2}{4} = 2$

(7) $\frac{x+4}{5} - x + 5 = \frac{x+3}{3} - \frac{x-2}{2}$

(8) $14\frac{1}{2} - \frac{2(m+3)}{5} = \frac{3m}{2} - \frac{2(m-7)}{3}$

43. (1) $\frac{1}{2}(z+1) + \frac{1}{3}(z+2) = 3 - \frac{1}{4}(z+3)$

(2) $x - \frac{3}{17}(2x-1) = \frac{7}{34}(1-2x) + \frac{10x-2}{2}$

(3) $\frac{2(2-3x)}{0{\cdot}01} - 2{\cdot}5 = \frac{0{\cdot}02-2x}{0{\cdot}02} - 7{\cdot}2$

(4) $\frac{3(1{\cdot}2-x)}{10} - \frac{5+7x}{4} = x + \frac{9x+0{\cdot}2}{20} - \frac{4(13x-0{\cdot}6)}{5}$

(5) $x - \frac{\frac{1-3x}{2}}{4} - \frac{2-\frac{x}{4}}{3} = 2$

(6) $1 - \frac{x-\frac{1+x}{3}}{3} = \frac{x}{2} - \frac{2x-\frac{10-7x}{3}}{2}$

44. Reduce to one term :

(1) $\frac{3}{x+y} - \frac{5}{x}$ (2) $\frac{4x}{5(x-3)} - \frac{3x}{2(x-3)}$

(3) $\frac{11a+13b}{3(a-b)} + \frac{15a+17b}{4(b-a)}$

(4) $\dfrac{1}{x-y} - \dfrac{1}{x+y}$ (5) $\dfrac{1}{3a-b} + \dfrac{1}{3a+b}$

(6) $\dfrac{x}{a-b} - \dfrac{x}{a+b}$

45. Simplify :

(1) $\dfrac{a}{3a+3b} - \dfrac{2a}{6a+6b}$ (2) $\dfrac{5b}{ax+ay} - \dfrac{2a}{bx+by}$

(3) $\dfrac{3m}{an+am} + \dfrac{2n}{bn+bm}$

(4) $\dfrac{x+4y}{2x(y+x)} - \dfrac{y-4x}{2x(y-x)}$

(5) $\dfrac{2x^2+x+10}{4x^2-25} + \dfrac{x}{5-2x}$

(6) $\dfrac{2z}{4-3z} + \dfrac{6z^2+2z-8}{9z^2-16}$

46. (1) $\dfrac{1}{a+2} + \dfrac{1}{a-2} - \dfrac{4}{a^2-4}$

(2) $\dfrac{b}{2a+2b} + \dfrac{a}{b-a} + \dfrac{a^2}{a^2-b^2}$

(3) $\dfrac{1}{c^2-cd} + \dfrac{1}{d^2-cd} + \dfrac{4}{c^2-d^2}$

(4) $\dfrac{5}{2x^2+6x} - \dfrac{4-3x^2}{x^2-9} - 3$

(5) $\dfrac{2}{n+2} + \dfrac{x+3}{n^2-4} - \dfrac{3x+1}{n^2-4n+4}$

(6) $\dfrac{3}{a+2} + \dfrac{a+1}{a^2-9} - \dfrac{a-1}{(a+3)(a+2)}$

47. Prove the following identities :

(1) $$\frac{2}{3a+6} - \frac{a-2}{2a^2+7a} - \frac{2}{3a^2+12a+12}$$
$$- \frac{4}{3a(a+2)^2} = \frac{1}{6a}$$

(2) $$\frac{1+x}{(x-y)(1-xz)} - \frac{1+y}{(x-y)(1-yz)}$$
$$- \frac{1+z}{(1-xz)(1-zy)} = 0$$

(3) $$\frac{ab+cd}{(a+c)(b-c)} + \frac{ac+bd}{(a+b)(c-b)} + \frac{ad+bc}{(a+b)(a+c)}$$
$$= 1$$

48. Simplify :

(1) $$\frac{2}{a-1} + \frac{5}{a+1} - \frac{3a}{(a+1)^2}$$

(2) $$\frac{1}{p-3} - \frac{3}{2p+6} - \frac{p}{12p^2-2p+18}$$

(3) $$\frac{1}{x-2a} + \frac{3}{x+2a} + \frac{8a^2}{4a^2x-x^4}$$

(4) $$\frac{4a^2-3a+5}{a^3-1} - \frac{1-2a}{a^2+a+1} + \frac{6}{1-a}$$

(5) $$\frac{3x+2}{x^2-2x+1} - \frac{6}{x^2-1} - \frac{3x-2}{x^2+2x+1}$$

(6) $$\frac{1}{a-b} - \frac{3ab}{a^3-b^3} - \frac{b-a}{a^2-ab+b^2}$$

(7) $$\frac{1}{y-5z}-\frac{z}{x^2+2xy}-\frac{x+y+5z}{xy-10yz-5xz+2y^2}$$

(8) $$\frac{a^2-bc}{a^2-ab+bc-ac}+\frac{3b-a}{2b-2a}+\frac{a+2c}{3a-3c}$$

(9) $$\frac{6a\,(a^2+8)}{a^3+64}+\frac{a^2-4}{a+4}-\frac{3a^2}{a^2-4a+16}$$

(10) $$\frac{x^2}{(x-y)(x-z)}+\frac{y^2}{(y-x)(y-z)}+\frac{z^2}{(z-x)(z-y)}$$

(11) $$\frac{1}{a\,(a-b)(a-c)}+\frac{1}{b\,(b-a)(b-c)}+\frac{1}{c\,(c-a)(c-b)}$$

(12) $$\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(a-c)^2}{(a-b)(b-c)}+\frac{(a-b)^2}{(c-a)(b-c)}$$

49. Solve the following equations :

(1) $a-\frac{x}{b}=c$ (2) $a\,(x+b)=c$

(3) $m\,(1+x)=n$ (4) $ax-bx=c$

(5) $$\frac{x}{a-b}+\frac{a^2-x}{a}=a+b$$

(6) $$\frac{x-kn}{k-n}-\frac{x}{k}=n$$

50. Calculate by the easiest method :

(1) $$\frac{59^2-38^2-97\times 11}{61^2-36^2}+\frac{56^2-26^2}{66^2-16^2}$$

(2) $$\frac{1}{2}+\frac{53^3-27^3}{31^2-25}-\frac{53^3+27^3}{58^2-22^2}$$

Solve the following problems :

51. If a car is bought for Rs. 15,600 and depreciates 20 per cent in the first year and 15 per cent in each subsequent year ; find its value at the end of three years.

52. If the cost of living increases by 10 per cent during a period of one year, by $12\frac{1}{2}$ per cent during the next year and 5 per cent during the following year, express the cost at the end of each year by means of 'index number' (given to the nearest unit), taking the value at the beginning at Rs. 100.

53. Two astronauts, Pat and Mike, were orbiting round the earth in separate space capsules. They were orbiting in the same direction and in the same plane. Pat orbits in 3 hours and Mike in $7\frac{1}{2}$ hours At 12 noon, Delhi time, Mike sees Pat directly below. At what time will Pat and Mike be one above the other again ?

54. 5 men or 8 women can reap the crop of a farm in 20 days. In how many days will 5 men and 8 women finish the job ? What will be the time taken by (*i*) 10 men and 8 women, (*ii*) 10 men and 16 women.

55. If 3 men or 7 boys can do a piece of work in x days, in how many days can the work be completed by (*i*) 3 men and 7 boys, (*ii*) 6 men and 7 boys, (*iii*) 3 men and 14 boys, (*iv*) 6 men and 14 boys ?

56. If x men or y women or z boys can paint a wall in 6 hours ; find the time in which the wall can be painted by (*i*) x men and y women (*ii*) y women and z boys (*iii*) x men, y women and z boys.

57. A mason can build a wall in x days ; but with the assistance of another mason he completes the work in y days. How long will it take the assistant to build the wall single handed ? If $x = 12$, $y = 8$; what will be the answer then ?

Form equations and solve :

58. One-third of a number is added to its one-fifth gives 16. Find the number.

59. When a number is divided by 7, the quotient is 3 and the remainder is 4. Find the number.

60. A number is multiplied by 4 and then 6 is subtracted from the product. The remainder when being divided by 18 gives 3 as quotient. Find the number.

61. The numerator and the denominator of a fraction are in the ratio of 6 : 11. If 3 is subtracted from numerator and the denominator the fraction becomes equal to $\frac{1}{2}$. Find the original fraction.

62. Ahmed covered $\frac{1}{3}$ of his journey by car, $\frac{1}{4}$ by taxi and the rest by train. If by train he covered a distance of 50 km, find the total distance of his journey.

63. A solution contains 60 grammes of sugar and 300 grammes of water. How much sugar must be added to the solution so that the resulting solution may be half sugar and half water ?

64. The numerator of a fraction is less than its denominator by 7. If 2 is added to each of the numerator and the denominator, the resulting fraction reduces to $\frac{1}{2}$. Find the fraction,

65. The denominator of a fraction exceeds its numerator by 18. If 7 is subtracted from each the numerator and the denominator, the fraction becomes zero. Find the fraction.

66. A man spends $\frac{1}{3}$ of his income on food, $\frac{1}{4}$ on his children's education and $\frac{7}{48}$ on other miscellaneons items. The rest he deposits in the post-office. Find what fraction of his income is deposited in the post office ?

7. Multiplication of Fractions

The rule for multiplication of the algebraic fractions is the same as that for the arithmetical fractions.

In order to multiply fractions, it is sufficient to multiply their numerators and denominators separately then the first product will be the numerator and the second product the denominator of the resulting fraction.

Example 6

Find the product of $\frac{a}{b}$ and $\frac{c}{d}$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$$

A whole expression can be considered a fraction the denominator of which is equal to 1, this rule can be used, even when the multiplicand is a whole expression.

67. Find the products :

(1) $\frac{15}{28} \cdot \frac{2}{3}$ (2) $\frac{3ab}{4xy} \cdot \frac{10x^2y}{21a^2b}$

(3) $\frac{4a}{15c^2} \cdot 3$ (4) $35x^4 \cdot \frac{8a^2}{7x^3}$

(5) $32x^3y^2 \cdot \frac{y}{48x^3}$ (6) $\frac{1}{x} \cdot y$

68. Simplify :

(1) $-\frac{9}{16} \cdot \frac{c}{a^4}$ (2) $\frac{b}{a} \cdot \left(\frac{-3a}{5}\right)$

(3) $\frac{-25x^4y^3}{14a^2} \cdot \left(\frac{-21ab}{10x^4y^2}\right)$ (4) $8a^2b^4 \cdot \left(\frac{-3a}{4b^3}\right)$

(5) $-3m \cdot \frac{n}{12m}$ (6) $-50a^2b^2 \cdot \left(\frac{-b^3}{125a^2}\right)$

69. Find the products :

(1) $\frac{6xy}{5ab} \cdot \frac{3ab}{4yx} \cdot \frac{4bx}{3axy}$ (2) $\frac{6m^3n^2}{35p^4} \cdot \frac{49n^4}{m^6p^3} \cdot \frac{5m^4p^7}{42n^6}$

(3) $\left(\frac{-32a^8b^6}{45c^{10}}\right) \cdot \left(\frac{55a^2c^9}{24b^4}\right) \cdot \left(\frac{27c^4}{22a^9b}\right)$

(4) $\left(\frac{-51a^6x^9}{56y^{11}}\right) \cdot \left(\frac{-40xy^8}{9a^5}\right) \cdot \left(\frac{-21y^3}{85ax^{10}}\right)$

70. Simplify :

(1) $\frac{a^2 - ab}{b} \cdot \frac{b^2}{a}$ (2) $\frac{a^2 - b^2}{a^2} \cdot \frac{a^4}{(a+b)^3}$

(3) $-\frac{(a+b)^2}{(a-b)^2} \cdot \frac{3(a-b)^2}{4(a+b)^3}$

(4) $\frac{5m-5n}{4m+4n} \cdot \frac{8m+8n}{15m-15n}$

(5) $\dfrac{ax+ay}{x^2-2xy+y^2}\cdot\dfrac{2x+2y}{ax^2+2axy+ay^2}$

(6) $\dfrac{2a^3-2b^3}{3a+3b}\cdot\dfrac{6a^2-6b^2}{a^2-2ab+b^2}$

(7) $\dfrac{5x^2-10xy}{x^2+4y^2}\cdot\dfrac{x^4-16y^4}{15(x-2y)^2}$

(8) $\dfrac{a^2-10ax-b^2+25x^2}{4a^3-ab^2}\cdot\dfrac{b-2a}{a+b-5x}$

(9) $\dfrac{n^2-10pk-2pn+5kn}{6kn-pn}\cdot\dfrac{p^2-6pk}{n^2-25k^2}$

(10) $\dfrac{x^4-y^4}{4x^2-2x-y-y^2}\cdot\dfrac{2x+y}{x^3+x^2y+xy^2+y^3}$

§ 8. Division of Fractions

If we consider the operation of division as an inverse operation with respect to the operation of multiplication, we shall obtain the following rule for division of fractions :

In order to divide a fraction by another fraction, it is sufficient to multiply the fraction of the dividend by the reciprocal of the fraction of the divisor :

Example 7

Divide $\dfrac{a}{b}$ by $\dfrac{c}{d}$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

Conversely, if we multiply the quotient obtained by the divisor, we shall obtain the dividend :

$$\frac{ad}{bc}\cdot\frac{c}{d}=\frac{a}{b}$$

This rule can be used, even if the dividend and divisor are whole numbers.

71. Find the quotients :

(1) $5a \div \frac{15a}{b}$ (2) $\frac{12xy}{25z} \div 8x^2$

(3) $\frac{8c}{21d^2} \div \frac{6c^2}{7cd}$ (4) $(-10a^2b^3) - \frac{5a^2}{b}$

(5) $36a^4b^5 \div \left(\frac{-12a^5}{b^6}\right)$ (6) $-\left(\frac{42x^4}{25y^3}\right) \div (-14x^3y^2)$

72. Find the results :

(1) $\frac{a^2b^3}{16c^4} \div \frac{a^3b^2}{80c^5}$ (2) $\frac{a^5x^4}{10c^9} \div \frac{a^4x^5}{54c^8}$

(3) $\frac{144a^{10}y^8}{65b^4z^3} \div \frac{60a^9y^7}{13b^3z^5}$ (4) $\left(\frac{8b^2cd}{9a^5} \div \frac{7cd}{12a^3}\right) \cdot \frac{28a^4}{3b^2}$

(5) $\frac{3p^2mq}{2a^2b^2} \cdot \frac{3abc}{8x^2y^2} \div 9a^2b^2c^3$

(6) $\frac{2ax}{yz} \div \frac{3bx}{ay} \div \frac{9b^2z}{8a^2xy}$

73. Simplify :

(1) $\frac{x^2 - y^2}{6x^2y^2} \div \frac{x + y}{3xy}$

(2) $\frac{-a^3 - 2a^2b}{72b^8} \div \frac{2b^9 - ax^3}{4b^6}$

(3) $\frac{2x^3 - 3xy^3}{160y^2} \div \left(\frac{-3y^5 - 2x^2y^3}{400y}\right)$

(4) $\frac{5 - 5a}{(1 + a)} \div \frac{10 - 10a^2}{3 + 3a}$

(5) $\frac{x^2 + xy}{5x^2 - 5y^2} \div \frac{x^2 - xy}{3x^3 + 3y^3}$

(6) $\frac{a^4 - x^4}{a^3 - x^3} \div \frac{a^2 + x^2}{a^2 - x^2}$

(7) $\frac{25a^6}{6a^3 - 6b^3} \div \frac{10a^4b}{9a^2 + 9ab + 9b^2}$

(8) $\frac{x^2 + 6xy + 9y^2 - 4z^2}{9x^2 - 4z^2} \div \frac{x^2 + 3xy + 2xz}{2z^2 - 3xz}$

(9) $\frac{15ab - 15 + 9a - 25b}{9a^2 - 30ab + 25b^2} \div \frac{9a^2 - 30a + 25}{15a - 25b + 9a^2 - 25b^2}$

(10) $\frac{x^3z + 125z}{x^2 - 16z^2} \div \frac{x^3 - 25x}{x^2 - 8xz + 16z^2} \cdot \frac{x + 4z}{x^2 - 5x + 25}$

74. Simplify the following fractions, using the basic property of a fraction :

(1) $\dfrac{\frac{1}{8}}{1 - \frac{3}{8}}$ (2) $\dfrac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2} + \frac{1}{3}}$ (3) $\dfrac{\frac{a}{b} - 1}{\frac{a}{b} + 1}$

(4) $\dfrac{\frac{2x - y}{5} - a}{\frac{2x + y}{5} + a}$ (5) $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}}$ (6) $\dfrac{\frac{a}{b} + \frac{b}{a}}{\frac{a}{b} - \frac{b}{a}}$

(7) $\dfrac{a - \frac{x^2}{a}}{x - \frac{a^2}{x}}$ (8) $\dfrac{\frac{1}{1 - x} + \frac{1}{1 + x}}{\frac{1}{1 - x} - \frac{1}{1 + x}}$

Solution of Q. No. 74, (5)

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}} = \frac{\frac{y+x}{xy}}{\frac{y-x}{xy}}$$

$$= \frac{y+x}{xy} \times \frac{xy}{y-x}$$

$$= \frac{x+y}{y-x}$$

Example 8

Find the integral part of the fraction : $\frac{2b+3c}{b+c}$

$$\frac{2b+3c}{b+c} = 2 + \frac{c}{b+c}$$

Integral part is 2.

75. Find the integral part in the following fractions :

(1) $\frac{x+2y}{x+y}$ (2) $\frac{1+a^2+a}{1+a}$

(3) $\frac{2+a+a^2}{1+a^2}$ (4) $\frac{3+x+x^2}{3+x^2}$

§ 9. Raising the Fraction to any Power

We know that :

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{2}{3} \cdot \frac{2}{3} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{3}{5}\right)^3 = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{3^3}{5^3}$$

$$= \frac{27}{125}$$

Therefore, in order to raise a fraction to any natural powe index, it is enough to raise the numerator and the denominator o the fraction to this power.

Example 9

Show that : $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (n is a natural number).

Solution

$$\left(\frac{a}{b}\right)^n = \frac{a}{b} \cdot \frac{a}{b} \ldots\ldots n \text{ times}$$

$$= \frac{a \,.\, a \,.\, a \ldots n \text{ times}}{b \,.\, b \,.\, b \ldots n \text{ times}}$$

$$= \frac{a^n}{b^n}$$

76. Evaluate the following fractions ·

(1) $\left(\frac{1}{2}\right)^4$ (2) $\left(\frac{2}{3}\right)^4$ (3) $\left(\frac{x}{y}\right)^5$

(4) $\left(\frac{5a}{4b}\right)^2$ (5) $\left(\frac{a^6 b}{cd^2}\right)^2$ (6) $\left(\frac{5c^2 d^2}{3a^3 b^3}\right)^3$

77. Simplify :

(1) $\left(\frac{-\,n^6 k^3}{10p^4}\right)^2$ (2) $\left(\frac{-\,a^2}{b^3}\right)^2$

(3) $\left(\frac{-\,a}{x^3}\right)^n$ (4) $\left(\frac{-\,b^2}{c}\right)^{2k-1}$

(5) $\left(\frac{-9x^7y^6}{a^{12}}\right) \cdot \left(\frac{a^8}{3x^5y^4}\right)^3$

(6) $\left(\frac{-2a^8b^3}{c^7}\right)^5 \div \left(\frac{-4a^{10}b^4}{c^9}\right)^4$

Example 10

Simplify :

$$\left(\frac{a}{a+1}+1\right)\div\left(1-\frac{3a^2}{1-a^2}\right)$$

Solution

$$\left(\frac{a}{a+1}+1\right)\div\left(1-\frac{3a^2}{1-a^2}\right)$$

$$=\left(\frac{a+a+1}{a+1}\right)\div\left(\frac{1-a^2-3a^2}{1-a^2}\right)$$

$$=\left(\frac{2a+1}{a+1}\right)\times\left(\frac{1-a^2}{1-4a^2}\right)$$

$$=\frac{(2a+1)}{(a+1)}\times\frac{(1-a)(1+a)}{(1-2a)(1+2a)}$$

$$=\frac{(2a+1)\times(1-a)\times(1+a)}{(a+1)\times(1-2a)\times(1+2a)}$$

$$=\frac{(2a+1)\times(1-a)\times(a+1)}{(a+1)\times(1-2a)\times(2a+1)}$$

$$=\frac{1-a}{1-2a}=\frac{a-1}{2a-1}$$

78. Simplify :

(1) $\left(\frac{a+1}{2a-2}+\frac{6}{2a^2-2}-\frac{a+3}{2a+2}\right)\left(\frac{4a^2-4}{3}\right)$

(2) $\left(\frac{9n^2-16m^2}{7n}\right)\left(\frac{3m-4n}{4m-3mn}-\frac{3m+4n}{4m^2+3mn}\right)$

(3) $\left(\frac{2}{a}-\frac{4}{a+2}\right)\left(2+\frac{a^2+4}{a-2}\right)$

(4) $(x^2-1)\left(\frac{1}{x-1}-\frac{1}{x+1}-1\right)$

(5) $\left(x - \frac{x+y}{x-y} + y\right) \div \left(1 - \frac{2y+1}{x^2 - y^2}\right)$

Example 11

Make the indicated operations and find the result :

$$\left(\frac{x^2}{y^2} + \frac{y}{x}\right) \div \left(\frac{x}{y^2} - \frac{1}{y} + \frac{1}{x}\right)$$

Solution

$$\left(\frac{x^2}{y^2} + \frac{y}{x}\right) \div \left(\frac{x}{y^2} - \frac{1}{y} + \frac{1}{x}\right)$$

$$= \left(\frac{x^3 + y^3}{y^2x}\right) \div \left(\frac{x^2 - xy + y^2}{y^2x}\right)$$

$$= \frac{x^3 + y^3}{y^2x} \times \frac{y^2x}{(x^2 - xy + y^2)}$$

$$= \frac{(x+y)(x^2 - xy + y^2)}{y^2x} \times \frac{y^2x}{(x^2 - xy + y^2)}$$

$$= x + y$$

79. Make the indicated operations and find the result :

(1) $\left(1 + \frac{a}{x} + \frac{a^2}{x^2}\right)\left(1 - \frac{a}{x}\right)\left(\frac{x^3}{a^3 - x^3}\right)$

(2) $\left(\frac{x}{x-a} - \frac{a}{x+a}\right) \div \left(\frac{x+a}{a} - \frac{x-a}{x}\right)$

(3) $\left(\frac{b}{a^2 - ab} + \frac{a}{b^2 - ab}\right)\left(\frac{a^2b + ab^2}{a^2 - b^2}\right)$

(4) $\left(\frac{m-n}{mn} - \frac{3m+n}{mn - m^2} + \frac{3n+m}{mn - n^2}\right) \div \left(\frac{2m+2n}{mn} + \frac{2m}{n-m}\right)$

(5) $\frac{b}{x-b} - \frac{ab}{x-a}\left(\frac{x+a}{ax - ab} + \frac{x+b}{b^2 - bx} + \frac{x}{ab}\right)$

80. Simplify :

(1) $$1 \div \left[\frac{a}{a-b} + \frac{4a^2b - 2ab^2}{b^3 - a^3} + \frac{b}{a^2 + ab + b^2}\right] - \frac{3ab}{(a-b)^2}$$

(2) $$\left(\frac{1}{2a-b} + \frac{3b}{b^2 - 4a^2} - \frac{2}{2a+b}\right) \div \left(\frac{4a^2 + b^2 + 1}{4a^2 - b^2}\right)$$

(3) $$\left(m + \frac{1}{1-m}\right) \div \left(m - \frac{m^2}{m-1}\right)$$

(4) $$\left(\frac{p}{p^2 - p} + \frac{2}{2-p} + \frac{1}{p+2}\right) \div \left(p - 2 + \frac{10 - p^2}{p+2}\right)$$

81. Prove the following identities :

(1) $$\frac{2}{mn} \div \left(\frac{1}{m} - \frac{1}{n}\right)^2 - \frac{m^2 + n^2}{(m-n)^2} = -1$$

(2) $$\left(\frac{2 - 2y}{3xy + 6y^2} - x^2 + 2xy\right) \times \frac{x + 2y}{x^2 - 2xy} + \frac{6xy^2 - 1}{3xy} = -x$$

(3) $$\left[\frac{p^2 - q^2}{pq} - \frac{1}{p+q} \cdot \left(\frac{p^2}{q} - \frac{q^2}{p}\right)\right] \div \frac{p-q}{p} = \frac{p}{p+q}$$

(4) $$\left[\frac{a-b}{(a+b)^2} - \frac{1}{a-b} \div \frac{a^2}{(a+b)(a-b) - 4ab} - \frac{b^2}{(a-2b)^2 - 5b^2}\right] \div \frac{8ab^2}{a^4 - b^4} + \frac{2b^2}{b^2 - a^2} = 1$$

Example 12

Solve the following equation :

$$\frac{1}{10x - 1} + \frac{1}{5x - 2} = 0$$

Solution

Multiplying both the sides of the equation by $(10x-1)(5x-2$ we get :

$$\frac{(10x-1)(5x-2)}{10x-1} + \frac{(10x-1)(5x-2)}{5x-2} = 0$$

$$\therefore \quad 5x - 2 + 10x - 1 = 0$$

$$15x - 3 + 3 = 0 + 3$$

$$15x = 3$$

$$\frac{15x}{15} = \frac{3}{15}$$

$$x = \frac{1}{5}$$

82. Solve the following equations :

(1) $\frac{3}{z-2} + \frac{7}{z+2} = \frac{10}{z}$

(2) $\frac{3t-1}{3t+1} = 2 - \frac{t-3}{t+3}$

(3) $\frac{y+5}{3y-6} - \frac{1}{2} = \frac{2y-3}{2y-4}$

83. Would the following equations be equivalent .

(1) $x - 2 = 5$ and $\frac{x-2}{x-3} = \frac{5}{x-3}$

(2) $x + 7 = 2$ and $\frac{x+7}{x+4} = \frac{2}{x+4}$

84. Solve the following equations :

(1) $\frac{2x-1}{2x+1} = \frac{2x+1}{2x-1} + \frac{8}{1-4x^2}$

(2) $\frac{t^2-3}{1-t^2} + \frac{t+1}{t-1} = \frac{4}{1+t}$

(3) $\dfrac{z+2}{z-2} = \dfrac{z^2}{z^2-4} - \dfrac{4}{2+z}$

(4) $\left(\dfrac{x-1}{x+3}\right)^2 = \dfrac{x+1}{x+9}$

(5) $\dfrac{4}{y^3-8} - \dfrac{1}{(y-2)^3} = \dfrac{3}{y^3+2y^2+4y}$

(6) $\dfrac{2}{(1-3x)(3x+11)} - \dfrac{1}{(3x-1)^2} - \dfrac{3}{(3x+11)^2}$

Example 13

Solve the following equation treating (*i*) x as unknown, (*ii*) y as unknown :

$$\frac{6-5x}{5-6x} = \frac{2y+3}{3y-2} - \frac{1}{2}$$

Solution

Multiplying both sides of the equation by $2\,(5-6x)(3y-2)$ we get :

$$2\,(6-5x)(3y-2) = 2\,(2y+3)(5-6x) - 1\,(5-6x)(3y-2)$$

$$2\,(18y - 15xy - 12 + 10x) = 2\,(10y + 15 - 12xy - 18x) - (15y - 18xy - 10 + 12x)$$

$$36y - 30xy - 24 + 20x = 20y + 30 - 24xy - 36x - 15y + 18xy + 10 - 12x$$

$$20x - 30xy + 36y - 24 = -36x - 12x - 24xy + 18xy + 20y - 15y + 30 + 10$$

$$20x - 30xy + 36y - 24 = -48x - 6xy + 5y + 40$$

$$x\,(20 - 30y) + 36y - 24 + 24 - 36y = -48x - 6xy \quad 5y + 40 + 24 - 36y$$

$$x\,(20 - 30y) + 48x + 6xy = -48x - 6xy + 48x + 6xy - 31y + 64$$

$$x\,(20 - 30y + 48 + 6y) = -31y + 64$$

$$x\,(68 - 24y) = 64 - 31y$$

$$\frac{x\,(68 - 24y)}{(68 - 24y)} = \frac{64 - 31y}{68 - 24y}$$

$$x = \frac{64 - 31y}{68 - 24y}$$

85. Solve the following equation treating (*i*) x as unknown (*ii*) y as unknown :

$$\frac{5y + 8}{y - 3} = \frac{3x - 7}{x + 2}$$

Example 14

Solve the following equation :

$$\frac{a}{x} - 1 = \frac{b}{x} - 9$$

Solution

$$\frac{a}{x} - 1 = \frac{b}{x} - 9$$

Multiplying both sides by x, we have

$$a - x = b - 9x$$

$$a - x - a = b - 9x - a$$

$$-x + 9x = b - 9x + 9x - a$$

$$8x = b - a$$

$$\frac{8x}{8} = \frac{b - a}{8}$$

$$x = \frac{b - a}{8}$$

86. Solve the following equations :

(1) $\dfrac{a-bm}{mx} - \dfrac{c-bn}{nx} = 1$

(2) $\dfrac{a}{a-x} = \dfrac{b}{b-x}$

(3) $\dfrac{a+b}{x-a} - \dfrac{a-b}{x+a} = 0$

(4) $\dfrac{x-s}{2x+t} - \dfrac{3x+t}{6x-s} = 0$

(5) $\dfrac{x+a}{2} - \dfrac{2}{x+a} = \dfrac{x-a}{2}$

(6) $\dfrac{m}{x+1} - \dfrac{m-1}{x} = \dfrac{1}{x-m}$

87. Solve for a :

$$S = \frac{(a+b)}{2}$$

88. Solve the equation :

$$\frac{1+a}{1-a} = \frac{b}{c} \text{ for } (i)\ a,\ (ii)\ b \text{ and } (iii)\ c.$$

89. The numerator of a fraction is less by 3 than its denominator. If the numerator is increased 3 times and the denominator is increased by 20 the fraction becomes $\frac{1}{8}$. Find the original fraction.

Solution

Let the denominator be x (a whole number)

$\therefore$ numerator $= x - 3$

and therefore fraction $= \dfrac{x-3}{x}$

In the second case the numerator

$$= 3(x - 3) = 3x - 9$$

and denominator $= x + 20$

$\therefore$ Fraction $= \dfrac{3x - 9}{x + 20} = \dfrac{1}{8}$,

Solving this equation, we have

$$x = 4$$

$\therefore$ Fraction $= \dfrac{1}{4}$

90. The denominator of a fraction is greater by 4 than its numerator. If numerator is increased by 11 and the denominator is decreased by 1; the fraction becomes inverse of the original fraction. Find the original fraction.

91. One of the two unknown numbers is greater by 12 than the other. If the smaller number is divided by 7 and the greater by 5, the first quotient is less than the second by 4 Find these numbers.

92. Distance between two towns A and B is 50 km. A cyclist started from town A to town B. After an hour and a half a person on a scooter also started in the same direction from town A and reached town B an hour earlier than the cyclist. Find the speed of each of them if the speed of the scooter is 2·5 times that of the cyclist.

93. Two trains start from the same station one after the other. The speed of the first train is 36 km/h and that of the other is 48 km/h. If the first train had started 2 hours earlier than the second, find when the second train would take over the first.

94. The distance between two stations can be covered by an electric train in an hour and a half. If the speed of the train is increased by 10 km/h the same distance can be covered in one hour and 20 mts. Find the distance between the stations.

95. The distance between two airports is 950 km. Two planes—a two engine plane and a rective plane—start simultaneously from these ports in opposite direction to meet each other. After half an hour the distance between these planes was 150 km. Find the speed of each plane if the speed of the rective plane is 3 times that of the two engine plane.

96. The smaller wheel of a vehicle make 15 revolutions more than the bigger wheel to cover a given distance. The circumference of the smaller wheel is 2·5 m and that of the bigger is 4 m. How many revolutions does each wheel make and what is the distance which the vehicle covers ?

97. Two wheels of a vehicle are connected by a leather strap. The circumference of the bigger wheel is 60 cm and that of the smaller is 35 cm. How many revolutions per minute does the smaller wheel make if the bigger wheel makes 84 revolutions per minute ?

98. A work was planned to be completed within 16 days. Six masons were employed on it. They finished the work two days earlier but every day they did the work of 2 masons more. How many masons can finish the work according to plan ?

99. Two labourers prepared a ditch for an electric station in 24 days The first labour takes $1\frac{1}{2}$ times longer than the other to do this work. Find the time taken by each to do the work separately.

100. In order to prepare 290 tools two workmen worked together for 4 hours and after this the first workman worked alone for $3\frac{1}{2}$ hours to finish the work. If they had worked together for 5 hours, the rest of the work would have been finished by the second worker in $2\frac{1}{2}$ hours How many tools per hour can each workman make ?

101. In order to type 71 pages two typists worked together for 4 hours and then the second typist worked alone for $2\frac{1}{2}$ hours more. If both the typists had worked together for $4\frac{1}{4}$ hours the second typist would have finished the work in 45 minutes. How many pages per hour can each typist type ?

102. The combined area of two plots which are sewn with corn is 120 ha. The crop of the first plot is 89 tonnes per ha and of the other is 95 tonnes per ha. Find the area of each plot if the crop of the first plot is greater by 1480 tonnes than that of the other.

103. A plank is floating on the waters of a river. Two boys weighing 50 kg and 30 kg are standing one on each of the two ends of the plank. If the length of the plank of 4 m, find the point at which the plank will be balanced.

104. The weight of an alloy of gold and silver is 1·06 kg. The weight of this alloy in water is 0.70 kg less. Find the weight of each metal in this alloy if gold loses its weight by $\frac{1}{19}$ and silver by 0·1 in water.

105. A piece of ice is floating in the sea. The volume of that part of ice which is above water is 2000 cc. If the weight of 1 cc of sea water is 1·03 g and weight of 1 cc of ice is 0·98 g find the volume of the piece of ice.

106. Eight adults can prepare a certain number of tools in 6 days and the same number can be prepared by 12 boys in 10 days. Find the number of days taken by 3 adults and 25 boys to prepare the same number of tools. What will be the number of days to prepare double the original number of tools ? What will be the time to prepare the same number of tools if the number of adults and also of the boys is doubled ?

107. Two pipes can fill a cistern in 6 and 8 hours separately. In what time the same cistern will be filled by both of them together if it is already one third full ? What will be the time taken by both the pipes to fill together an empty cistern of double the previous size ?

108. Gopi takes half as much long again as Nanak takes to finish a piece of work. Find the time taken by Gopi to finish the same work, one-third of which has already been finished by Nanak in an hour.

109. A man can whitewash $\frac{5}{3}$ times faster than a woman, if both of them can whitewash a house in 6 days ; find the time taken by an woman to do the work all alone.

Coordinates and Prime Diagrams

§ 1. Coordinates of a Point on the Plane

1. Find the distance of the place S from the railway track and from the motor-road (Fig. 6 1), if the scale is 1 square = 10,000 m.

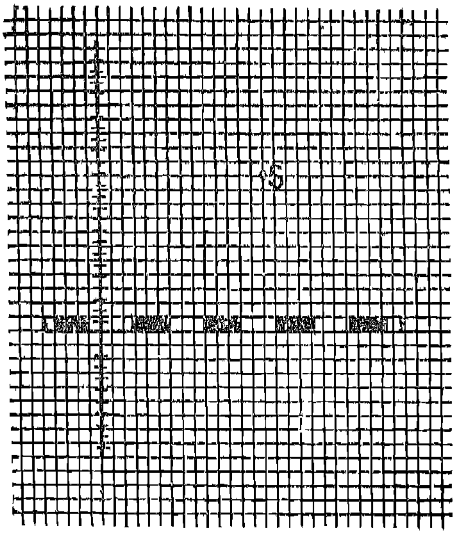

Fig. 6 1

Draw two mutually perpendicular lines XOX′ and YOY′ (Fig. 6.2).

We call these lines *axes of coordinates.* The first axis XOX′ (in the horizontal position in the figure) is called axis of X or X-axis. The second axis YOY′ (in the vertical position in the figure) is called axis of Y or Y-axis. Point O, the point of intersection of the axes is called the *origin of the coordinates* or simply *origin.* We shall consider this point as zero point or the starting point.

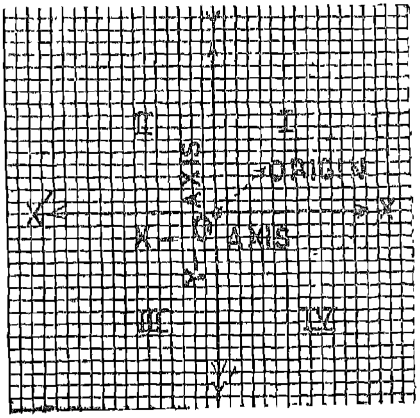

Fig. 6 2

On the axis of X, positive numbers are represented to the right and on the axis of Y these are represented above the origin. The negative numbers are represented to the left of the origin on the axis of X and below the origin on the axis of Y.

Plane on which these axes of the coordinates are shown, is called the *coordinate plane.*

The axes of coordinates divide the plane into four regions (parts), which are called *quadrants.* It is customary to denote these quadrants by Roman digits as marked in Fig. 6.2.

2. Find the distances of any point say A, from the axes of coordinates (see Fig. 6 3).

Take a point M on the plane and draw the perpendiculars to the axes of coordinates through the point M.

Let the point M_1 be the foot of the perpendicular MM_1 drawn from M to X-axis (Fig. 6.4). The number which is represented by

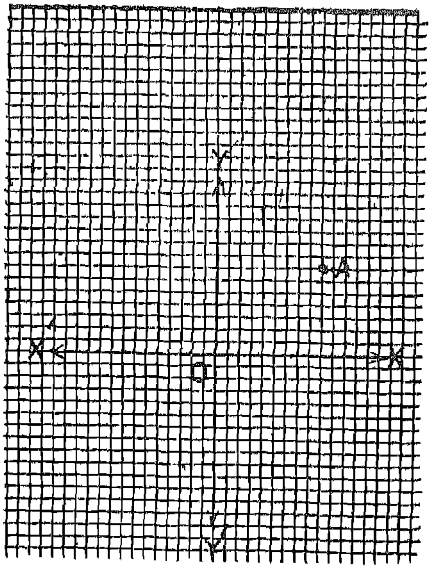

Fig. 6.3

Scale: 1 div. = 1 unit.

he point M_1 (or the measure of the distance OM_1) on the X-axis is called the *abscissa* or the x-coordinate of the point M.

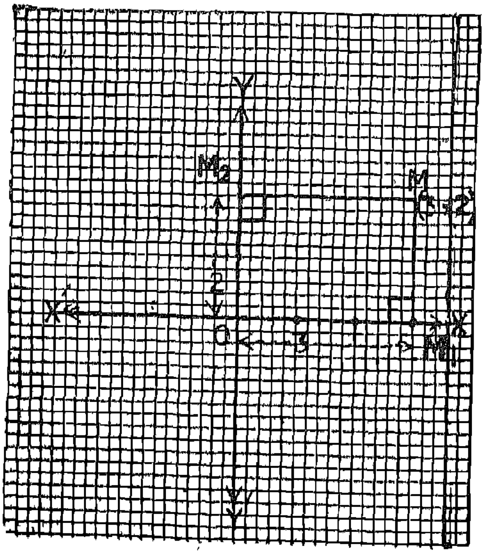

Fig 6.4

Scale : 5 div. = 1 unit.

Let the point M_2 be the foot of the perpendicular MM_2 to Y-axis. The number which is represented by the point M_2 (or the measure of the distance OM_2) on the Y-axis is called the *ordinate* or the *y*-coordinate of the point M. It is why the axes of coordinates X-axis and Y-axis are also called the axis of abscissa and the axis of ordinate. In Fig. 6.4, the abscissa of the point M is 3, and its ordinate is 2.

Abscissa and ordinate of a point are called the coordinates of the point. These denote the distances of the point M from the axes of Y and X respectively which are equal to the actual distances of the point M, viz., MM_2 from Y-axis and MM_1 from X-axis.

3. Determine and write down the coordinates of each point marked in Fig. 6.5.

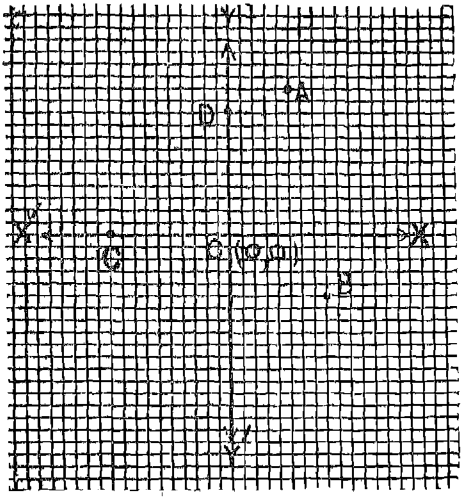

Fig. 6.5

Scale : 5 div. = 1 unit.

The coordinates of a point are written in brackets to the right of the point. So the symbol M (3, 5) means that 3 is the abscissa of M and 5 is the ordinate of M.

Let us consider different positions of a point on the coordinate plane.

If a point lies on the axis of abscissa, its ordinate is equal to 0.

For example, the point C has coordinates 2 and 0 (Fig. 6 5).

If a point lies on the axis of ordinates its abscissa is equal to 0.

For example, the point D has coordinates 0 and 2.

Finally, origin—the point O, has abscissa and ordinate both equal to 0 expressed as (0, 0).

Now let us solve the following examples :

Example 1

Locate a point P on the paper if its coordinates are given, for example let us locate the point P (2, -3).

Solution

The point is obviously in the fourth quadrant. Find the point corresponding to number 2 on the axis of X and draw perpendicular to XOX′ through this point. Find the point corresponding to number -3 on the axis of Y and draw the perpendicular to YOY′ through this point (Fig. 6.6). The point of intersection of these perpendiculars is the point P, which we wanted to locate.

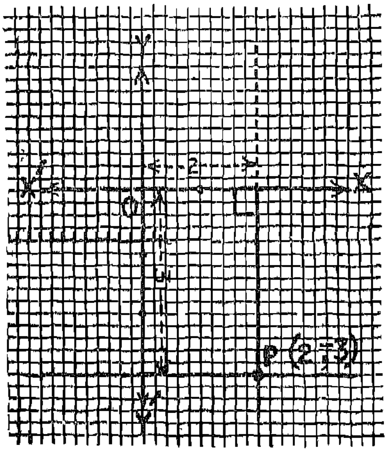

Fig. 6 6

Scale : 5 div. = 1 unit.

Thus we see that we can locate only one point on the plane when its coordinates are given.

It means that a pair of ordered numbers determines the position of a point on the plane.

Therefore, the coordinates of a point are numbers, which determine its position on the plane.

Example 2

The axes of coordinates and any point K are given on the plane. Find the coordinates of K.

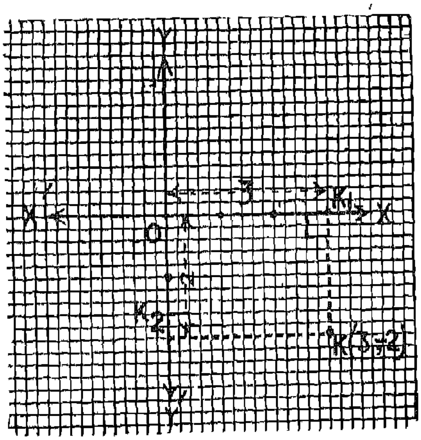

Fig. 6.7

Scale : 5 div.=1 unit.

Solution

Draw through the given point K, the perpendiculars KK_1 and KK_2 respectively to the axes XOX′ and YOY′ (Fig. 6.7). The feet of these perpendiculars will determine the coordinates of K. In our case these are ($^{+}3$, $^{-}2$).

We can denote the abscissa and ordinate of a point by the letters x and y also.

4. Locate the points having the following coordinates.

(1) $x = 5, \quad y = 3$	(2) $x = {}^{-}4, \quad y = 6$
(3) $x = {}^{-}3, \quad y = {}^{-}4$	(4) $x = 5, \quad y = {}^{-}2$

(5) $x = -2,\quad y = 0$ (6) $x = 0,\quad y = 3$
(7) $x = 0,\quad y = 0$ (8) $x = 0,\quad y = -5$

5. Without locating the positions of the points determine the quadrants in which the following points lie :

(1) $x = 3$, $y = 5$ (2) $x = -3$, $y = -5$ (3) $x = 2$, $y = -7$
(4) $x = -5$, $y = 1$ (5) $x = 0$, $y = -1$ (6) $x = -1$, $y = 0$
(7) $x = 0$, $y = 0$

6. Construct the triangle ABC, if the coordinates of its vertices are A (4, 5) ; B (8, 0) and C (-6, 3).

7. Construct the point A_1 symmetrical to the point A (4, 6) with respect to the axis of X. Find the coordinates of A_1. Compare the ordinates and the abscissas of A_1 and A.

8. Construct the point B_1 symmetrical to the point B (-2, 3) with respect to the axis of Y.

Find the coordinates of B_1.

Compare the ordinates and the abscissas of B_1 and B.

9. Construct the point D_1 symmetrical to the point D (-3, 2) with respect to the origin.

Find the coordinates of the point D_1 from the graph. Compare the ordinates and the abscissas of the points D and D_1.

10. The following points are situated on a plane :

(1) A (1, 3) (2) B (2, 5) (3) C (1, -3)
(4) D (-2, -5) (5) E (-1, 3)

Find the points symmetrical to these points with respect to X-axis, Y-axis and the origin.

11. (1) Construct a quadrilateral ABCD, with vertices A (0, 0) ; B (1, 3) ; C (8, 5) and D (9, 1).

(2) Calculate the area of this quadrilateral having divided it by diagonals (first by one diagonal, then by the other).

(3) Calculate the arithmetical mean of two results thus obtained and round up the result upto two digits.

12. The results of measurements of the temperature of air during a day are given in the following table :

Time of the day in hours	0	2	4	6	8	10	12	14	16	18	20	22	24
Temperature in °C	-2	0	-3	-5	0	4	6	9	6	3	1	0	-2

(1) Draw the graph of the changes of the temperature during the day.

(2) Read from the graph, the temperature of the air at 3 hours, at 9 hours, at 13 hours and at 21 hours.

(3) Find from the graph the time at which the temperature was -1° ; -4° ; +2° and +5°.

(4) Find the time of the highest and the lowest temperature during the day.

13. Represent graphically the dependence between x and y. Use the following tables :

(*a*)

x	-4	-5	-2	-1	0	1	2	3	4		
y	-2	-0·8	0	0·3	1	2	3·2	3·8	4·2		

(*b*)

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	4	2	0·5	-0·4	-0·6	-0·8	-0·2	0·8	1	0·6	0·2

(1) For what values of x in the table above, the value of y is negative, positive, and 0 ?

(2) For what values of x, the value of y increases/decreases, has the greatest value, has the least value ?

14. Prepare the table of dependence between x and y with the help of the graph of the dependence given below :

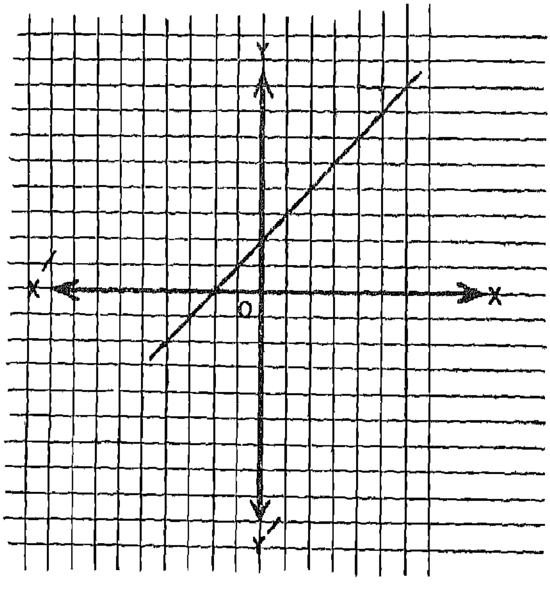

Fig. 6.8
Scale : 1 div.=1 unit.

§ 2. Direct Proportional Dependence

We studied directly proportional values in arithmetic. For example, the distance and the time of covering the distance at a uniform speed are directly proportional values.

Let the speed of a uniform motion be 3 km/h. If we denote the distance by 'y' and the number of hours for which this body is in motion by 'x' then the dependence between these values of distance and time will be expressed by :

$$y = 3x$$

Another example of directly proportional values will be the total cost of articles and the number of articles. Let the cost of article be Rs. 8·00. If we denote the number of articles to be purchased by 'x' and the total cost by 'y', the dependence between these two values can be expressed by :

$$y = 8x$$

We can give many other examples of directly proportional values. The length of the circumference of an article and the length of its diameter is an example from geometry. Dependence between the length of the circumference and the length of the diameter is expressed by the formula :

$$y = 3{\cdot}14x \text{ (approximately)}$$

(x is the length of the diameter and y is the length of the circumference).

Let us consider the following equations :

$$\frac{y}{x} = 3 ; \frac{y}{x} = 8 ; \frac{y}{x} = 3{\cdot}14$$

We can see that the corresponding values of x and y are directly proportional and their ratio is always the same.

Therefore, if two numbers x and y are directly proportional, then every value of y will be equal to the corresponding value of x multiplied by the ratio of $y : x$. Such dependence between two numbers is called the **direct proportional dependence.**

Definition : Dependence between two numbers x and y, which is expressed by the formula $y = kx$ ($k \neq 0$), is called direct proportional dependence.

Number k is called the '**coefficient of proportionality**'. In the above mentioned examples these coefficients are 3, 8 and 3.14 respectively. It is clear that x and y can be any numbers, positive and negative but not zero.

15. One kg of grapes cost Rs. 4.00.

(1) Work out a formula, which can express dependence between y the cost of the grapes and the quantity x kg bought.

(2) Draw a graph of the formula obtained.

(3) Determine from the graph the cost of :
2 kg 500 g ; 4 kg 250 g ; 3 kg 750 g of grapes.

(4) Determine from the graph the quantity of the grapes which can be bought for Rs. 7, Rs. 11 and Rs. 9.

(5) How will the change in total cost of grapes y depend on the change in quantity x purchased ?

16. (1) Knowing that there is direct proportional dependence between y and x, fill the following table :

x	8	6	4	2	1	$\frac{1}{2}$	0	-1	-2	-4	-6	-8
y	4											

(2) Write down the formula of dependence between y and x.

(3) Draw the graph of this dependence.

§ 3. Graph of Direct Proportional Dependence

In the previous problems, the graphs of proportional dependence were drawn. It is not difficult to check up with the help of a ruler that all the points marked are situated on the same line which passes through the origin of the coordinate axes.

A set of the points on the plane, which satisfies the formula $y = kx$, is called a graph of direct proportional dependence.

And so, the graph of direct proportional dependence is a straight line passing through the origin of the coordinate axes. As a straight line is determined by the positions of two of its points, so in order to draw a straight line, it is enough to know the positions of any two points lying on it.

One of these points will be the origin which is already fixed. Therefore, we have to mark the position of only one point of the straight line for drawing the graph of direct proportional dependence.

17. There is a direct proportional dependence between y and x and the coefficient of proportionality is equal to 4.

(1) Write down the formula of dependence between y and x.

(2) Fill in the following table :

x	2	5	7	$\frac{1}{2}$	0·25
y					

(3) Construct a graph of these dependences.

18. Construct the graph of the dependences $y = 3x$ and $y = -3x$ on the same figure. Compare the graphs of these dependences.

19. Construct the graph of the following dependences having the same coordinate axes :

$$y = \frac{1}{3}x\,; \qquad y = -\frac{1}{3}x$$

$$y = 2x\,; \qquad y = -2x$$

Compare the graphs of these dependences.

20. Fill in the following table, being given that there is direct proportional dependence between y and x :

x	y	x	y	x	y	x	y	x	y
3	12	2			21	8	4	4	6
4		1				10		10	
5		6	12	3			7		24
7		5		4	20	5		20	

21. The height of a child changes upto 10 year in the following way :

Age in years $=y$	1	2	3	4	5	6	7	8	9	10
Height in cm $=h$	70	80	88	95	100	107	113	119	127	129

(1) Show that the ratio of any of the values of h is not equal to the ratio of the corresponding values of y.

(2) Show that ratio of any of the values of h to the corresponding values of y is not equal to the same number.

(3) Can we say that this dependence is direct proportional dependence ?

(4) Construct the graph of this dependence. What kind of lines will these be ?

§ 4. Linear Dependence

Example 3

A group of pupils started from a town. Now their distance from the town is 5 km and they are going with a uniform speed of

3 km per hour. What will be the distance from the town after x hours ?

Solution

In x hours the pupils will cover $3x$ km and 5 km were covered earlier. Therefore in x hours, the distance from the town will be equal to $(3x + 5)$ km. Let us denote this distance by the letter y.

Then we shall have

$$y = 3x + 5$$

This equality expresses dependence between the distance and the time. This dependence is not the direct proportional dependence as the ratio of the distance to the time is not equal to the same number or is not a constant (see the table below) :

$x=$	1	2	3
$y=$	8	11	14

Definition : Dependence between two values of x and y which is expressed by the formula y = ax + b, where a and b are numbers, is a linear dependence.

If $b = 0$, then $y = ax$.

Therefore, direct proportional dependence is a particular case of linear dependence.

Let us construct the graph of the dependence $y = 2x + 3$. Firstly we shall construct the graph of dependence $y = 2x$. This is a straight line passing through the origin.

Let us see how the point of the graph $y = 2x + 3$ is situated with respect to the straight line $y = 2x$.

For this we shall work out the following table :

x	$^-2$	$^-1$	0	1	2	3
$2x$	$^-4$	$^-2$	0	2	4	6
$2x+3$	$^-1$	1	3	5	7	9

We can see that for one abscissa, the ordinate of the point of the second graph is greater than the respective ordinate of the point of the first graph by 3.

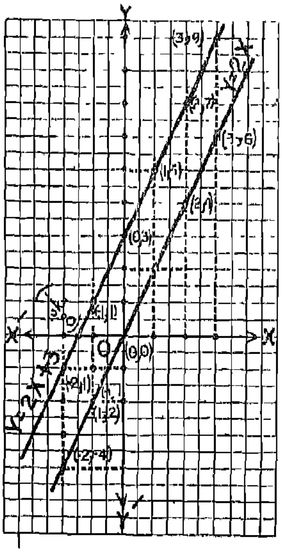

Fig. 6.9

Scale : 2 div.=1 unit.

Therefore, the ordinate of any point of the second graph is greater than the ordinate of the corresponding point of the first graph by 3 units. Having constructed the graph of dependence $y = 2x + 3$, we shall obtain the straight line parallel to the straight line $y = 2x$ (Fig. 6.9).

Conclusion :

The graph of linear dependence is a straight line. In order to construct the graph of linear dependence, it is enough to find any two of its points.

Usually for constructing this graph find the two points of its intersection with the axes of coordinates.

So, if $x = 0$, $y = 3$

and if $x = \frac{-3}{2}$, $y = 0$

Therefore the graph of the dependence $y = 2x + 3$ will pass through the points A (0, 3) and B $\left(\frac{-3}{2}, 0\right)$

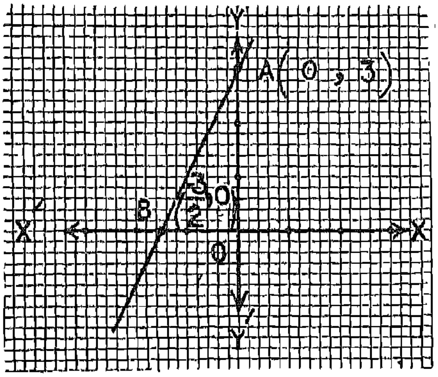

Fig. 6.10

Scale : 5 div.=1 unit

22. At 8.00 hours the temperature was 6°F. Upto 16.00 hours the temperature rose uniformly by 0·5°.

(1) Write the formula of dependence between the temperature and the time.

(2) Construct the graph of this dependence.

(3) Find on the graph, the temperature at 10.00 hours; at 12.00 hours and at 13.30 hours.

(4) Can we say that the temperature and the time are directly proportional ?

(5) How is the dependence $y = 0{\cdot}5x + 6$ called ?

23. A train starts from a station A and after travelling 2 km it assumes a uniform speed of 40 km.

(1) Find, how far away from the station A the train will be after 0.5 hour of uniform motion.

(2) Calculate the values of s (distance in kilometers) for the following values of t (time in hours) :

t hours	0·1	0 2	0·3	0·4	0·5	0 6	0·7	0·8	0·9	1
s km										

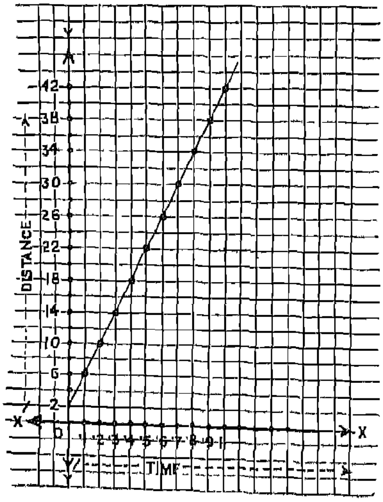

Fig. 6.11

Scale : 1 div =·1 hr. 1 div.=2 km.

(3) Construct the graph of dependence between s and t (Fig. 6.11).

(4) What are the points of difference and similarity between the graph thus obtained and the graph of direct proportional dependence ?

(5) Verify by calculation that it is not direct proportional dependence.

24. Construct the graphs of the dependences on the same figure after having filled the following table :

x	-4	-3	-2	-1	0	1	2	3	4
$y=\frac{1}{2}x$									
$y=\frac{1}{2}x-2$									

(1) What difference is there between the positions of these straight lines with respect to the axes of coordinates ?

(2) Show that the straight lines $y = \frac{1}{2}x$ and $y = \frac{1}{2}x - 2$ are parallel.

(3) Find the length of the segment, cut on the axis of Y by the straight line $y = \frac{1}{2}x - 2$.

25. (1) Construct the graphs of the dependences $y = 2x + 1$ and $y = -2x + 1$ on the same figure.

(2) What is the difference between these graphs ?

(3) Show that if x increases, the value of y in $y = 2x + 1$ also increases uniformly and the value of y in $y = -2x + 1$ uniformly decreases.

26. (1) For the graphs of the dependences $y = 2{\cdot}5x - 10$ and $y = -1{\cdot}5x + 3$, find such values of x, for which $y > 0$ (or y is a positive number).

(2) For the graphs of the dependences $y = 4{\cdot}2x - 6$ and $y = -2x - 8$, find such values of x, for which (*i*) $y = 0$ and (*ii*) $y > 0$.

27. (1) Find the lengths of the segments cut on the positive sides of the axes by the straight line $y = 1{\cdot}5x - 6$.

(2) The straight line $y = kx + 10$ cuts on the axis of X (from the origin to positive direction) a segment equal to 2·5 units in length. Find the coefficient k.

28. (1) Construct the graphs of the following dependences on the same figure :

$$y = -\frac{1}{2}x - 4 \qquad y = \frac{1}{2}x - 4$$

$$y = -\frac{1}{2}x + 4 \qquad y = \frac{1}{2}x + 4$$

Find the points of intersection of these graphs with the axes OX and OY.

(2) Find the angular coefficients for each of these straight lines.

(3) Calculate the value of x, when $y = 3$ (for each dependence) and check up the results on the graphs.

§ 5. Inverse Proportional Dependence

We studied the inverse proportional values in the course of arithmetic.

For example, the length of the base of a rectangle and its width, when the area of the rectangle is constant, are inverse propartional values. We can verify this from the following table :

Length in *m*	20	30	40	50	60
Width in *m*	30	20	15	12	10
Area in m^2	600	600	600	600	600

If we denote the length of a rectangular plot by x and the width by y, then the dependence between them can be expressed by the formula :

$$xy = 600 \quad \text{or} \quad y = \frac{600}{x}$$

Time and speed of uniform motion, when the distance is constant, are also inverse proportional values.

Let us work out the following table :

Speed in km	10	20	40	50	80
Time in hours	20	10	5	4	$2\frac{1}{2}$
Distance in km	200	200	200	200	200

If we denote the speed by x and time by y, we can express the dependence between speed and time by the formula :

$$xy = 200$$

Definition : Dependence between the two values of x and y expressed by the equality xy = k where k, is any number (not equal to zero) is called inverse proportional dependence. Number k is called coefficient of proportionality.

Values of x and y can be any numbers except zero. Indeed, if one of these values is equal to zero, the left hand side of the equality $xy = k$ becomes equal to zero and the right hand side of the equality is not equal to zero. And thus we shall obtain a false equality.

Let us construct the graph of an inverse proportional dependence, for example, the graph of the dependence $xy = 9$.

Having expressed y in x, we shall obtain $y = \frac{9}{x}$

We shall put permissible values for x and obtain corresponding values of y and work out the following table :

x	-9	-6	-3	-2	-1	$+1$	$+2$	$+3$	$+6$	$+9$
y	-1	$-1\frac{1}{2}$	-3	$-4\frac{1}{2}$	-9	$+9$	$+4\frac{1}{2}$	$+3$	$+1\frac{1}{2}$	$+1$

Let us plot the corresponding points on the coordinate plane.

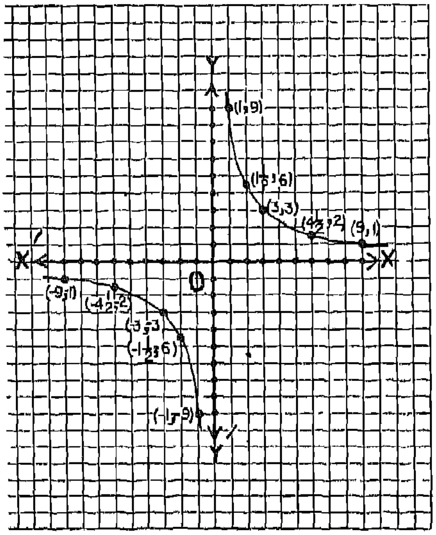

Fig. 6.12

Scale : 1 div.=1 unit.

If we take more values for x, then the points will be situated closer and closer to each other. We can see in Fig. 6.12, that the graph of the dependence $xy = 9$ is a curve line. This line has two parts (branches) which are symmetrical with respect to the origin (one branch for positive values of x and the other for negative values of x).

One branch is situated in the I quadrant and the other in the III quadrant.

The graph of the inverse proportional dependence is called a *hyperbola.*

In Fig. 6.13, the graph of the dependence $y = -\frac{9}{x}$ (with negative coefficient) is drawn.

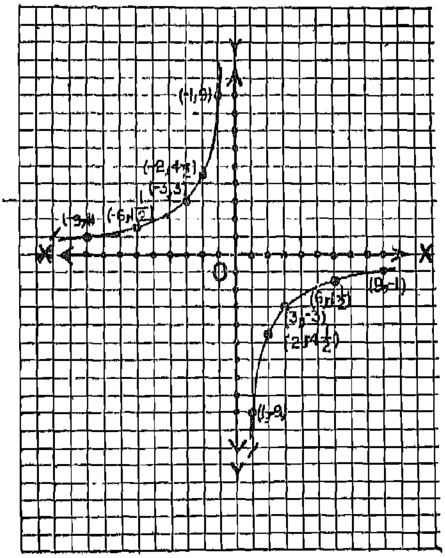

Fig. 6.13

29. The area of a rectangle is 12 cm^2 and its length is a cm.

(1) Find the width of this rectangle having denoted it by the letter b.

(2) Calculate the width from the dependence $b = \frac{12}{a}$ for the

following values of a :

Length in cm $a=$	1	2	3	4	5	6	8	12
Width in cm $b=$								

Using the table of values for a and b answer the following questions :

(3) Prove that the width of the rectangle is inversely proportional to its length when the area is constant.

(4) Construct the graph of this dependence.

(5) Using the graph, find the values of b, if $a = 1{\cdot}5$; $a = 2{\cdot}4$; $a = 9$.

(6) Find from the graph, the values of a, if $b = 10$; $b = 5$; $b = 8$.

(7) Will the graph be crossed by the coordinate axes ?

(8) On the same figure construct rectangles with $a = 1$; $a = 2$; $a = 3$; $a = 4$; $a = 6$ and prove that the area of each rectangle is equal to 12.

30. Construct the graph of the dependence $y = -\frac{8}{x}$ after having filled the following table :

$x=$	-8	-5	-4	-2	-1	1	2	3	4	5	8
$y=$											

Answer the following questions :

(1) What values can x have ?

(2) What will be the corresponding values of y ?

(3) How will y change if x changes ?

(4) What kind of symmetry does the graph have ?

31. Construct the graphs of the dependences $y = \frac{4}{x}$ and $y = -\frac{10}{x}$.

32. Fill the following table and write the equation of the dependence, knowing that the values of x and y are inversely proportional and the coefficient of proportionality is -9.

x	-9	-6	-4		-1		4	6			18	36
y				-18		12			36	3		

33. Write an equation of inverse proportional dependence for the graphs represented in Fig. 6.14 and 6.15.

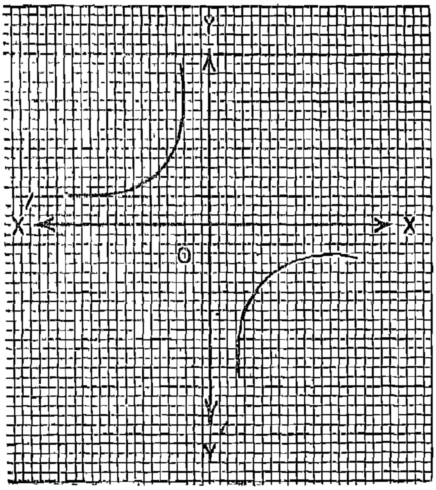

Fig. 6 14

Scale : 1 div.=1 unit.

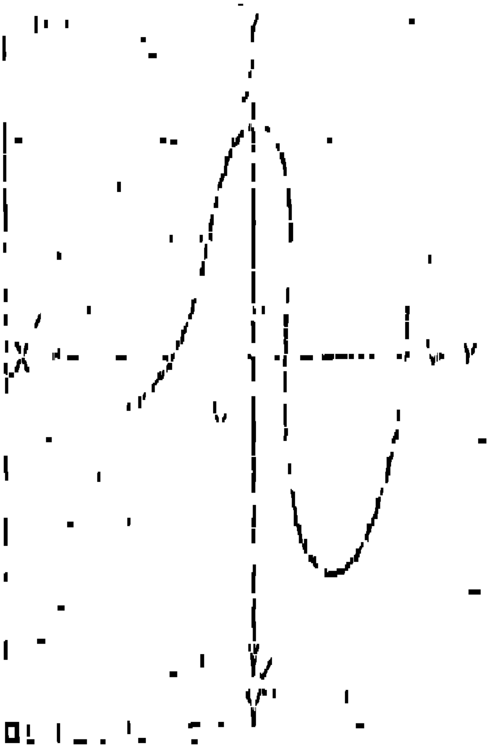

Fig. 6.15

Scale : 1 div.=1 unit.

Answers

CHAPTER I

1. All except $\frac{2}{0}$ and $\frac{a}{b-b}$ **5.** (1) Infinite (2) Infinite (3) Infinite

6. (1) Infinite (2) Infinite (3) Infinite **9.** Yes **10.** No

11. (*a*) 3 (*b*) $5, {}^-5$ (*c*) (1) 6, (2) $6, {}^-6$

12. (*a*) 3, Infinite (*b*) ${}^-3$ (*c*) $5, {}^-5$; $10, {}^-10$; no

13. $a=1, 4, 6, 8, 10\ldots\ldots$ $b=1, 2, 3, 4, 5\ldots\ldots$

14. If a is a multiple of b then $\frac{a}{b}$ is a whole number

17. (1) false (2) false (3) true (4) true

18. (1) ± 2 (2) ± 0.3 (3) $\pm\frac{5}{6}$ (4) $3, {}^-5$

(5) ${}^-\frac{1}{6}, \frac{7}{6}$ (6) ${}^-0{\cdot}9, 1{\cdot}5$

19. (1) 3, 1 (2) $\frac{1}{2}$ (3) 3·4 (4) ± 2 (5) $\frac{5}{4}$ (6) 1·6

20. (1) 0, 1 (2) 0, 1 (3) 0 **21.** Yes.

24. (1) $\frac{8}{3}$ (2) $\frac{44}{13}$ (3) 14·987 (4) 1·456

25. (1) $\frac{31}{15}$ (2) ${}^-\frac{17}{12}$ (3) 1.136 (4) ${}^-\frac{13}{60}$ (5) $\frac{1}{4}$

26. (1) $-\frac{2}{5}$ (2) $-3{\cdot}5$ (3) $\frac{a}{b}$ (4) 0 **27.** No **28.** No

29. (1) $\frac{7}{23}$ (2) 11·2 (3) $\frac{a}{b}$ (4) 0

30. (1) 0·5 (2) 2·905 (3) 6412·326 (4) ${}^-3{\cdot}82$ (5) ${}^-0{\cdot}09$ (6) 0

31. (1) $\frac{7}{4}$ (2) ${}^-\frac{11}{4}$ (3) $\frac{1}{9}$ **32.** (1) $\frac{ad-bc}{bd}$ (2) $\frac{ps-qr}{qs}$

34. (1) $\frac{17}{6}$ (2) ${}^-5\frac{3}{7}$ (3) ${}^-12{\cdot}7$ (4) 6·9 (5) $-\frac{e}{f}$ (6) $\frac{x}{y}$

35. (1) $1\frac{3}{8}$ (2) 2·4 (3) $\frac{^-37}{19}$ (4) $\frac{32}{27}$

36. (1) $\frac{15}{2}$ (2) $^-2.22$ (3) $\frac{1}{3}$ (4) 16 8

37. (1) $7\frac{1}{2}$ (2) $\frac{16}{9}$ (3) 65 (4) 8·9

38. (1) $\frac{^-3}{2}$, $\frac{5}{7}$, $\frac{1}{4}$, $\frac{d}{c}$, $\frac{1}{x}$, $\frac{x}{3}$, $\frac{1}{px}$, $\frac{1}{7.9}$, No reciprocal

39. $^-1$, $\frac{x}{y}$, 1, 0 **40.** (1) $\frac{2}{3}$ (2) 10 (3) $^-1$ (4) 1

46. (1) 0 (2) 0 (3) 0 (4) 0

47. (*a*) 12 m. (*b*) $p+30$ years (*c*) $\frac{m}{4}$ km/h (*d*) $5x$ cm ; $\frac{3}{2}l^2$ sq. cm

48. (*a*) $m=\frac{a+b+c+d}{4}$ (*c*) $d=S-5(a+b)$

(*d*) $a+b$; $a-b$, when $a=b$ boat stands still in water , when $a<b$; moves along the current with velocity $b-a$ km/h.

49. $a=bc$, no **50.** $F=\frac{9C}{5}+32$; 32°, 68°, 5°, $^-40°$

51. $a=4b+2$ **52.** $\frac{5\ 9}{10\ 4}$ or $\frac{59}{104}$

53.

x	0	$1\frac{1}{2}$	2	3	$^-1$	$^-2$
y	2	5	4	$3\frac{1}{2}$	$1\frac{2}{3}$	$2\frac{2}{3}$

54. (*a*) $\frac{a\times n}{100}$ (*b*) $\frac{100\times b}{p}$ **55.** $\frac{5m}{4}$ **56.** $\frac{500}{b}$ percent

57. $\frac{tm}{t-m}$ **58.** $44\frac{4}{9}\ p$, $40p$ **59.** 2 years **60** 46 5 kg.

61. 27 6 mm **62.** Rs. 117 **63.** 92·5% **64** $6\frac{1}{4}$%

65. 239 365 approximately **66** Rs. 500 **67.** $9\frac{1}{11}$ % **68.** Yes

69. 1·5 m **70.** Rs. 9·60 **71.** 9 hours **72.** Rs. 10·10

73. 15% **74.** 30 paise each **75** 8 paise per Re **76.** Rs. 3100

77. 30 years **78** Rs 200 **79** 3 years **80.** Rs 300

81. 3% **82.** 1% loss **83** Second **84.** $3\frac{1}{2}$ years

85. 10% **86.** 12%

CHAPTER II

1. Constant terms $-\frac{3}{4}$, 4 5, $^{-}5$, $\frac{7}{3}$ Variables x, y, z

2. Constant **3.** 0, 1, 2, 3, 4, **4** 2, 4, 6, 8 **5** 31, 37, 41, 43, 47

6. $S=a^2$, 1·44 cm^2 **7** $V=a^3$, 64 cm^3

8. 226981, $^{-}243$, 0.56733696, 128, 7 414875, 335 54432

9. 16 0 and 335 54432 , $^{-}15$ and $^{-}243$

10. :

Nos	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sq of Nos	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

11.

Nos	1	2	3	4	5	6	7	8	9	10
Cubes of Nos	1	8	27	64	125	216	343	512	729	1000

12. 10, 100, 1000, 10000, 100000, 1000000 ; number of zeros is equal to the index.

13. 0·1, ·01, ·001, ·0001, ·00001, ·000001, number of zeros between the decimal point and 1 is one less than the index.

14. (*a*) $10^2<2^{10}$ (*b*) $2^4=4^2$ (*c*) $3^4>4^3$

15. No, it is only true when $a=b$ or when $a=4$ or 2 and $b=2$ or 4

16. (*a*) 1, 1, 1, 1 ; any natural number to the power 1 gives 1

(*b*) 0, 0, 0, 0 ; any natural number to the power 0 gives 0.

17. $-$ve, $+$ve, $+$ve, $+$ve, $+$ve, $+$ve, $+$ve, $-$ve, neither $+$ve nor $-$ve

18. (*a*) 0 (*b*) $(^{-}2.4)^4$ (*c*) 7^4 (*d*) $(^{-}2)^4$ (*e*) 1^5

19. no **20.** when $a=0$ **21.** (*a*) yes (*b*) yes (*c*) no **22.** yes

23. ±2, ±3, ±4, ±5, ±6, ±7, ±8, ±9, ±10

24. (*a*) $^{-}500$ (*b*) $^{-}35$ (*c*) $^{-}125$ (*d*) $^{-}125$

26. (*a*) 15×10^7 km (*b*) 43×10^7 (*c*) $51\times10^7\ \text{km}^2$, $149\times10^6\text{km}^2$, $361\times10^6\text{km}^2$

(*d*) 72×10^5 (*e*) 27×10^{18} **27.** 3600000, 450000000, $14>1000000000$

28· $^{-}4.056$ **29.** (*a*) 36m^3 (*b*) $37{\cdot}268\text{m}^3$

30. $s = 44{\cdot}1$ m, $313{\cdot}6$ m, 122.5 m, 1960 m

31. $2(a^2+2ab)$, 1008 cm^2 **32.** (*a*) 001296 (*b*) $84{\cdot}5$ (*c*) 1 (*d*) 0

33. 300000 km/sec

34. (*a*) $3^3 < 3^5$ (*b*) $(^{-}3)^3 > (^{-}3)^5$ (*c*) $(0{\cdot}2)^3 < (0{\cdot}2)^5$ (*d*) $(^{-}0{\cdot}2)^3 > (^{-}0\ 2)^5$
(*e*) $(-3)^3 < (-3)^2$ (*f*) $3^5 > 5^3$

35. (*a*) when $0 < a < 2$, $2a > a^2$, when $a = 2$, $2a = a^2$
when $a > 2$, $2a < a^2$ (*b*) $b^6 > 0$

36. 9, two, $^{+}9$ and $^{-}9$ **37.** ± 4 **39.** $0{\cdot}2$

40. $y = {}^{-}2, {}^{-}1{\cdot}2, {}^{-}1, 0, 1\ 2$ **41.** 0 ; value of s will remain 4

42. $s = 6a^2$, $10{\cdot}14$ cm^2 **43.** $V = a^2h$ cm^3 $h = \dfrac{V}{a^2}$ m **44.** 3

45. $2x+3y-1$, $a+x+5$.........etc

46. $\dfrac{2x-1}{2}, \dfrac{4x}{7}, \dfrac{2x+3}{5}, \dfrac{2y-1}{3}$.... ...etc

49. Whole expressions $2a - 3$, $\dfrac{-2b}{3}$, $\dfrac{3x^2-4x+5}{3}$, $^{-}4 + \dfrac{3b}{5}$,

$2x - 3x + 5$, $x^2 + y^2$, $\dfrac{a^2 - ab}{15}$

Fractional expressions : $\dfrac{a+e}{4b}$, $\dfrac{-a}{2x-1}$, $3 - \dfrac{3}{5b}$, $\dfrac{-(a^3-b)^2+3}{7a}$

51. $7, 0{\cdot}5, \dfrac{2}{3}, 1, {}^{-}1, \dfrac{1}{3}, \dfrac{3}{5}, \dfrac{^{-}1}{2}$

52. $3a^2mx, {}^{-}5a^2x^2, -ab^2c, {}^{-}5bc^3, 2ma, 3m^2ax^3, -4x^2, \dfrac{5}{2}a^3bc$

$3, {}^{-}5, {}^{-}1, {}^{-}5, 2, 3, {}^{-}4, \dfrac{5}{2}$

53. (*a*) 4, (*b*) $^{-}80$, (*c*) $(^{-}32)$, (*d*) $^{-}24$

54. (*a*), (*b*), and (*e*) are monomials

55. (*a*) $1{\cdot}2$, (*b*) $^{-}0{\cdot}65$, (*c*) $6a$, (*d*) $11b^2$, (*e*) 1, (*f*) $^{-}1$ (*g*) $^{-}0{\cdot}3a$

56. $^{-}5a^3$, $15a^2x^2$, $^{-}8abx$, $0.2xy$

57. (*a*) $2a + 5x$, (*b*) $0\ 5b + (-y)$, (*c*) $(^{-}3ac) + (^{-}5xy)$

58. (*a*) $8y - 9ab$, (*b*) $3b + 0.2$, (*c*) $(^{-}7ax) - (ay)$ **59.** $3x^2y + 3b - c$

60. $^{-}5xy + 2x - 7m$ **62.** 1,2,5,6 parts are polynomials

63. (*a*) $(^{-}6x^5) + 3x^4 + (^{-}12x^2) + 5$
(*b*) $2.5x^4 + (^{-}18x^3y) + (^{-}16x^2y^2) + (^{-}3xy^3)$

64. (a) $\frac{^{-}11}{36}$, (b) 3 **65.** (a) $x^5 + 7ax^4 - 8a^2x^3 + 3ax^2 - 5a^3x$

(b) $- 0{\cdot}6a^2 + 0{\cdot}2a^2x^2y - \frac{3}{4}ax^3 + \frac{5}{6}xy^2$

66. $-8y + 0{\cdot}1bx^2 + \frac{5}{8}b^2xy^2 - \frac{5}{9}b^3x^2y^3$ **68.** $4a$ **69.** $p/3$

70. 1500 a litre **73.** Yes

74. $ax^2, -ax^2, 7abx^2, -0{\cdot}3abx^2, \frac{-2}{3}ab, ab$ **75.** (1) $^{-}11$m (2) 0

76. (1) $^{-}2m^3$ (2) $^{-}9k^4$ **77.** (1) x^2, (2) $2{\cdot}6n^6$ **78.** (1) $y^2 - y$

79. (1) $8ab - 6a^2b^3 - 8ab^2$ **80.** (1) $\frac{1}{15}a^2x + \frac{1}{10}ax^2$ (2) $-(x-y)$

81. (1) 4 (2) 18 **83.** (1) $8a$, (3) $-5d$,

84. (1) $^{-}1\frac{5c}{12}$ (2) $1{\cdot}7x^2$ **85.** (1) $\frac{1}{4}a^2b + \frac{2}{3}ax^2$ (2) $^{-}0{\cdot}08ac - 2{\cdot}7c$

86. (1) 60 **87.** 1·7 kg, 0·2 kg and 0·1 kg. **88.** 20 cm, 16 cm and 12 cm

89. 720 pages **90.** (1) $9a^n + 7a^{n+1}$, (2) $4x^{k+1} + x^k$

91. (1) 40, (2) $^{-}5$ **92.** (1) $^{-}25$ (2) 10

94. (1) $3{\cdot}5a + 2x$ (2) $\frac{10}{3} - 3y$ **95.** (1) $20a^2 + 2a$, (2) $3{\cdot}5a^2 + 1{\cdot}7b^2$

96. (1) $19a - 8b + c - 2d$ (2) $7x^2 - 3ax$

97. (1) $2a^4 - a^2 + 1$, (2) $b^6 + b^4 + 2b^2$, (3) $4a^n - 7b^m$ **98.** (1) 2

99. (1) $(17a^4 - 8a^3x) + (-6a^2x^2 - ax^3)$ (2) $(3y^5 - 12y^3z^2 - z^5) + (15y^4z + 2yz^4)$

100. (1) 2000, (2) 1200

101. (1) $2a^4 - 4a^3 - 6a + 7$, (2) $a^3 + b^3$, (3) $2a^3 - 8a^2b - 0{\cdot}2ab^2$

102. $^{-}5a^8 + 7a^6 - 3a^4 + 2a^2 + 10$ (2) $3b^5 - 4b^4 + 5b^3 - 6b^2 + 8$

103. $2b^3 + b^2c + 4c^3$ (2) $^{-}20x^4 + x^3 - 5x^2 + 10x + 1$ **105.** $14m + 6n$

106. $5 \times 10^3 + 3 \times 10^2 + 7 \times 10 + 2$, $4 \times 10^4 + 8 \times 10^3 + 9 \times 10^2$
$+ 3 \times 10 + 6$, $3 \times 10^5 + 9 \times 10^4 + 2 \times 10^3 + 7 \times 10^2 + 4 \times 10 + 5$

110. 1000, 2000, 1500 **111.** 4cm, 5cm, 6cm

112. (1) $11x$ (2) $2c^4$ (3) $-k$ (4) $7q$

113. (1) $^{-}0{\cdot}4a^2$ (2) $^{-}6a^{2n+1}$ (3) m^2n (4) $3a$

114. (1) $10bc^3$ (2) $^{-}13y^3$ (3) $^{-}18a^2b^7$ **115.** (1) $6{\cdot}9\,n^4p$ (2) $7{\cdot}1k^3n^4$

116. (1) 0 (4) $\frac{p}{5}$ **118.** $3m + 9n$ (2) $-5a^2 + 12a$

119. (1) $28x - 21y + 25z$ (2) $14ab - 53bc - 13cd$ (3) $3p^2 - 2p$
(4) $x^2 + y^2 + 1$

120. (1) $\frac{5}{6}x^2y^2 - \frac{3}{4}ab - \frac{11}{6}a^2b^2 - \frac{3}{4}$
(2) $^{-}\frac{7}{3}x^3 - \frac{8}{3}x^2y + \frac{9}{4}xy^2 - \frac{1}{2}y^2 - \frac{1}{3}$

121. (1) $-2{\cdot}4b^3b^2c - 2{\cdot}25a^4b^3c^2 + 1{\cdot}62a^5b^4c^3$ (2) $12{\cdot}15x^2 + 2{\cdot}865xy$

122. (1) $-7a^{n+3} - 2a^{n+2} + 10a^{n+1} - 3a^n$ (2) $y^{n+1} + y^n + y^{n-1} - y^{n-2}$

123. x^2 **124.** (1) $\frac{31}{4}$ (2) $9{\cdot}4$

125. $2x + (-y^3) - (^{-}5x^2y) - (^{+}4xy)$

126. (1) $(13x^6 + 7x^2 + 8x + 4) - (6x^4 + 5x^3)$ (2) $3y^2 - (y-1)$
other solutions are also possible

127. $(2a^2 + 4) - (6a)$ **128.** (1) $8a^2 - b^2$ (2) $9a^2 - 7b^2$ (3) $18m - 9n$

129. (1) 0 (2) 1. **130.** (1) $a+(3a - 2b)$ (2) $x - y + (x-y)$

131. 1*m*, 3*m*, 4·5*m* **132.** 57 cm. **133.** 75°, 50°, 55°

134. 96, 24 **135.** 21, 24 **136.** 16 km/p

137. 590 thousand km², 108 thousand km², 66 thousand km²

138. (1) $2a$ (2) $-2t$ (3) $\frac{4b - 2a}{3}$ **139.** $14{\cdot}7 - 15{\cdot}1y + 6y^2$

140. (1) $(^{-}2x - y^2) - (^{-}4x^2 + y)$ (2) $a^4 - 9 - (^{-}2a^3)$

141. (1) $-4a^2 - 7b^2 + 7c^2$ (2) $17a^2 - 22a + 16$ **142.** (1) 15 cm, 15 cm, 18 cm

144. (1) $^{-}4{\cdot}46$ (2) $^{-}240$ **145.** (1) c^{12} (2) 5^{15} (3) k^{10} (4) y^{2k+1}

146. (1) $^{-}2a^3$ (2) $0.6a^3xy$ (3) $^{-}0{\cdot}38$ (4) $6abc$

147. (1) $(-b)^{2n}$ (2) x^{4n+7} (3) 0 (4) 0

148. (1) $0{\cdot}09\,a^4x^2y^4$ (2) $^{-}1{\cdot}2\,k^5b^4$ (3) $^{-}20{\cdot}21\,b^3x^4$

149. (1) $10x^{m+3}$ (2) $(a - b)^9$ (3) $-(x - 2)^{10}$.

150. (1) $^{-}12a^5$ (2) $100x^{2n+2}y^2$ (3) $-x^{5n}y^{5m}$ (4) a^4 (5) $^{-}9a^2b^2$

151. (1) 132 (2) 12 **153.** (1) $zk - 2nk$ (2) $99a - 63c$ (3) $10a^2 + 15ab - 25ac$
(4) $12nc - 8nb + 28an$

154. (1) $3p^3q + \frac{3}{2}p^2q^2 - 2pq^3$ (2) $a^3b^2 - 2a^2b^3 - \frac{10}{9}ab^3$
(3) $^{-}2{\cdot}25\,x^2 + 1{\cdot}5\,xy - 2{\cdot}5y^2$ (4) 0

155. (1) $\frac{^{-}1}{2}p^{2m}q^{m-n} + 3p^{11}q^{m-n+4}$ (2) $2m^2n^x + 3m^xn^4$

156. (1) $a^2 + 2ab - b^2$ (2) $^{-}3a - 7b$ (3) $16 - 10{\cdot}5x$ (4) $4y - 23x + 18$

157. (1) $\frac{-1}{3}$ (2) $4\frac{1}{4}$ (3) $^{-}3$ (4) $^{-}12$

158. 8 m, 4 m **159.** 171 per day.

160. (1) 5P = 11 ; 3P = 15 (2) rabbits = 575 ; hens = 425

163. (1) $45mp + 18mq - 35np - 14nq$ (2) $12a^4 - 14a^2b^2 - 20b^4$
(3) $7x^4 - 13x^2y^2 - 24y^4$ (4) $2x^4 + 4x^3y - 13x^2y^2 - 6xy^3 + 15y^4$
(5) $x^4 - y^4$ (6) $b^4 - y^4$

164. (1) $3a^n + 3a^{n-1}b + ab^{n-1} + b^n$ (2) $a^{x+4} - a^2b^{x-2} - a^{x+2}b^2 + b^x$

165. (1) $2a^4 - 3a^3 - 101a^2 + 204a - 90$
(2) $2x^6 - 6x^5 + x^4 + 14x^3 - 18x^2 + 7x + 2$

166. (1) Constant term = 7, term coutaining highest power = $^{-}3a^4$
(2) Constant term = 0, term containing highest power = $3{\cdot}45x^{10}$

168. (1) $x^2 + 8x + 15$ (2) $n^2 + 7n - 18$ (3) $m^2 - 5m + 6$
(4) $x^2 - 11x - 12$

169. $439\frac{1}{2}$ **170.** (1) $\frac{-4}{5}$ (2) $3\frac{1}{7}$ **171.** 12m **172.** 360 sq cm.

173. 8, 9, 10 and 11 **175.** 24 m, 12 m

176. (1) $22x^2 - 9x + 3$ (2) $a^5 + a^4 - 3a^2 - a$

178. (1) $\frac{1}{2}$ (2) $-\frac{1}{5}$ **179.** 2 m **180.** 42 dm

181. (1) z^{5k} (2) x^{6k} (3) y^{8n} (4) $x^{4(k-1)}$

182. (3) c^{16} (5) c^{4n+2} (7) $y^{2n(2n-1)}$ (8) $-y^{6n}$

183. (1) $\frac{1}{9}b^6$ (2) $^{-}0{\cdot}027x^6z^3$ (3) $\frac{^{-}125}{8}x^{12}y^9$

(4) $^{-}5a^9b^6c^3$ (5) $\frac{^{-}1000}{27}a^{3n}b^6$ (6) $\frac{^{-}32}{27}x^9y^{12}$

184. (1) a^{3n-3} (2) $0{\cdot}0016\,a^{4(k-11)}x^{4(10-k)}$ (3) $0{\cdot}125\,x^{3(k+1)}x^{9k}$ (4) $a^{3(2k-15)}$

185. (1) not equal (2) equal (3) not equal (4) equal.

186. (1) $a^{10}b^{12}$ (2) $b^{10}c^{30}$ (3) $3a^{20}C^{18}$ (4) $\frac{125}{3}a^{10}x^{10}$ (5) $64a^{20}y^{16}$

187. (1) $m^2 + 2mk + k^2$ (2) $b^2 + 2bc + c^2$ (3) $9a^2 - 6ax + x^2$

(4) $4a^4b^2 + 2a^2bxy + \frac{1}{4}x^2y^2$

189. (1) $a^2 + 10a + 25$ (2) $9c^2 + 6c + 1$
(3) $9a^2x^6 + 48a^4x^4 + 64a^6x^2$ (4) $\frac{a^2}{16} + \frac{ab}{6} + \frac{b^2}{9}$
(5) $4m^2 + 6m + \frac{9}{4}$

190. (1) $a^{2m} + 2a^mb + b^2$ (2) $\frac{1}{4}a^{2(n-1)}\,b^4 + a^{2n}b^2 + a^{2\,(n+1)}$

191. (1) $5b$, $20ab$ (3) $4b$, $80ab$; $100a^2$

192. (1) $(a + 3)^2 + 4$ (2) $(x + 4)^2 + 5$ (3) $(2a + 3b)^2 + b^2$
(4) $(5 + b)^2 - 24$

193. (2) 10609 (3) 26·01 **194.** (1) -3 (2) -1 (3) 0

195. (1) $a^2 - 2ab + b^2$ (2) $4x^2 - 4xy + y^2$

196. (1) $c^2 - 2cd + d^2$ (2) $a^2 - 8a + 16$ (3) $9a^2 - 6ab + b^2$
(4) $0{\cdot}01\,b^2 - 0{\cdot}4b + 4$

198. (1) $a^{14}\,b^{10} - 2a^7b^{10} + b^{10}$ (2) $0{\cdot}04a^{10}\,x^6 - 2a^5x^3b^3y + 25b^6y^2$
(3) $0{\cdot}04y^4 - 0{\cdot}12y^5 + 0{\cdot}09y^6$ (4) $5{\cdot}76p^6 - 24p^3q^2 + 25q^4$
(5) $a^{2m} - 2a^{m+1} + a^2$ (6) $4x^2y^{2p-2} - 4xy^{p+3} + y^8$

200. (1) $1{\cdot}44x^4y^2 - 1{\cdot}2x^5y^3 + 0{\cdot}25x^6y^4$ (2) $6{\cdot}25m^4n^6 + m^5n^5 + 0{\cdot}04m^6n^4$
(4) $\frac{25}{9}a^{6n} + 4a^5 + \frac{36}{25}a^{2(5-3n)}$

201. $4ab$ **202.** (1) n^2 (2) a^2 (3) $10xp$ (4) $4a^4$

203. (1) $(x - 10)^2 - 97$ (2) $(x - 5)^2 + 2$ (3) $(a - 6)^2 - 37$
(4) $(3x - 5)^2 - 5$

204. (1) $7b$, $42bx$ (2) $3b$, $64a^2b^2$, $48ab^2$ **205.** (3) 998001 (4) 24·01

206. (1) $84x^2 - 90xy + 37y^2$ (2) $95a^2 - 570x^2$ (3) $45a^2 + 25b^2$
(4) $-9p^2 - 42pq + 126q^2$

208. (1) $\frac{3}{2}$ (2) 2 **209.** (1) $a^2 - c^2$ (2) $a^2 - 1$ (3) $4a^2 - b^2$
(4) $4d^2 - \frac{1}{4}$ (5) $b^4 - a^4$ (6) $\frac{1}{9}\,x^2 - 0{\cdot}25y^2$

210. (*a*) (1) $6\frac{1}{4}\,x^2 - \frac{9}{16}y^2$ (2) $0{\cdot}0004\,t^2 - 0{\cdot}25\,u^2$ (3) $\frac{1}{9}f^2 - 5{\cdot}29k^2$
(4) $a^8 - b^8$ (*b*) (1) 1596 (2) 9999 (3) 1400 (4) 1

212. (1) $-m^2 - 4m + 8$ (2) $-7 - 16x + 7x^2$ (3) $16x^4 - 81y^4$
(6) $\frac{1}{4}x^4 - 4y^3 + 4xy^2 - x^2y^2$

213. (1) 6 (2) 3 (3) $\frac{1}{3}$

214. (1) $1 + 3b + 3b^2 + b^3$ (2) $x^3 + 12x^2y + 48xy^2 + 64x^3$
(3) $27a^3x^3 + 27a^2x^2 + 9ax + 1$ (4) $a^3 + 3a^2b^3 + 3ab^6 + b^9$

215. (1) $64x^9 + 240x^6y^2 + 300x^3y^4 + 125y^6$
(2) $\frac{1}{27}a^3 + \frac{1}{6}a^2b + \frac{1}{4}ab^2 + \frac{1}{8}b^3$
(3) $0{\cdot}125x^3 + 0{\cdot}075x^2y + 0{\cdot}015\,xy^2 + {\cdot}001y^3$
(4) $0{\cdot}008a^9 + 0{\cdot}6a^5 + 0{\cdot}15a^3 + 0{\cdot}125a^3$
(5) $1000x^9 + 100x^6y^2 + \frac{10}{3}x^3y^4 + \frac{1}{27}y^6$
(6) $b^{3k} + 3b^{2k} + 3b^k + 1$ (7) $1 + 3a^{n+1} + 3a^{2n+2} + a^{3n+3}$
(8) $0{\cdot}001x^{3n-3} + 0{\cdot}03x^{2n-2}\,y^{n+1} + 0{\cdot}3x^{n-1}\,y^{2n+2} + y^{3n+3}$

216. (1) $x, 3ax^2, x^3$ (2) $1, 6a, 12a^2, 8a^3$ (3) $y^2, 8x^9, 6x^3y^4, y^6$

218. (1) 1 (2) 1 **219.** $5, 7, 9,$

221. (1) $27 - 27b + 9b^2 - b^3$ (2) $x^3 - 9x^3 + 27x - 27$
(3) $8a^3 - 36a^2b + 54ab^2 - 27b^3$ (4) $8a^9 - 24a^6b^2 + 24a^3b^4 - 9b^6$
(5) $\frac{x^3}{8} - \frac{x^3}{4} + \frac{x^3}{6} - \frac{x^3}{27}$
(6) $0{\cdot}001b^{12} - 0{\cdot}15b^{11} + 0{\cdot}075b^{10} - 0{\cdot}125b^9$

224. (1) $z, z^3, 3zb^2$ (2) $1{,}125b^3, 75b^2, 15b$ (3) $b^2, b^6, 3a^4b^2, a^6$

225. (1) ${}^{-}2$ (2) 1 (3) $\frac{1}{3}$

226. (1) $a^3 + 1$ (2) $1 + m^6$ (3) $8a^3 + 27$ (4) $n^3 + \frac{1}{8}$

227. (1) $b^3 - 1$ (3) $27a^3 - 64$ (4) $\frac{1}{8}a^3 - 8b^3$

228 (1) 6859 (2) 970299 (3) 1·030301 (4) 110·592

229. (1) $8\frac{1}{8}$ (2) ${}^{-}30$ **230.** (1) 14 (2) 11 (3) 3 (4) 3 (5) 18 (6) 76

232. (1) 2 (2) 3 **235.** (1) 9^3 (2) m^3 (3) c (4) p^4 (5) 1 (6) 0

236. (1) $-y^2$ (2) $-z^2$ (3) 1 (4) a (5) x^{m-n} (6) c^n

237. (1) $4xy$ (2) $2bd$ (3) $-3n$ (4) 1 **238.** (1) No. (2) No

239. (1) $4m^2n$ (2) $4p^3c$ (3) ${}^{-}3$ (4) ${}^{-}4ab$ (5) $4a^3b^2$ (6) $\frac{3}{2}cy^4$

240. (1) ${}^{-}\frac{3}{4}a^{m-2}\,b^{n-1}\,c^2$ (2) ${}^{-}5b$

241. (1) $-\frac{6}{5}b^9c$ (2) $\frac{1}{3}b^2$ (3) x^5 (4) 3^{10}

242. (1) $\frac{a}{6}$ (2) $\frac{8}{c}$ (3) $\frac{7b}{2c}$ (4) $\frac{5ab}{4pq}$

243. (1) $2a + 1$ (2) $4 - 5b$ **244.** (1) $ab + 1$ (2) $x - ax$ (3) $-3 + 5x^2y^2$
(4) $2x^2 + x + 4$ (5) $\frac{1}{2}ay - \frac{1}{2}y^3$ (6) $\frac{1}{4}x - \frac{1}{4}y$

245. (1) $1 - 3xy^2 + 4x^2y^3$ (2) $3a^2 + 4ax + x^2$ (3) $-8m^3 - 12m + 2$
(4) $-6n^3 - 9n^2 + 3n$ (5) $5p^3 + 20p^2 - 30p - 40$ (6) $-6m^3 - 6mn^2 + n^4$

246. (1) 12, $2a$ (2) $16x^4y$, $32x^6y^3$ (3) $70b^7c^{11}$, $1{\cdot}5bc^2$

247 (1) $-4a + 5$ (2) 3 (3) $a^2 - 2a - 2$ (4) $-x$

249. (1) $^-22{\cdot}8$ (2) $\frac{^-158}{15}$ **250.** (1) 3 (2) 5 (3) $^-4$

251. (1) $\frac{5}{3}b^2 - ab - 5a^2 + 4$ (2) $7a^2 - 0{\cdot}3ab - 2{\cdot}5\,b^2$

CHAPTER III

1. {6, 12, 18, 24, 30, 36, 42, 48}

2. {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 47}

3. {...$^-5$, $^-3$, $^-1$, 0, 1, 3, 5, 7, 9, ... ,999}

4. 34 **5.** (*a*) finite (*b*) infinite (*c*) finite (*d*) finite
(*e*) infinite (*f*) infinite (*g*) infinite

6. (*a*) {1, 4, 9, 16, 25, 36, 49...... .} (*b*) {1, 2, 3, 4, 5, ... 109}
(*c*) {Citizens of India} **7.** Set of even natural numbers. **8.** ϕ

9. (*a*) ϕ (*b*) ϕ (*c*) {24, 28, 32, 36} (*d*) {5, 10, 15, 20, 25.... ...}

10. (*ii*) and (*iii*), **11.** Open statements (*d*), (*e*), (*f*), (*h*), (*i*), (*j*)

12. (*a*) ϕ (*b*) {3}

13. (*a*) Two (0, $^-5$) (*b*) One (3), (*c*) Two (± 2) (*d*) No (*e*) Infinite

14. (*a*) and (*d*) **15.** (*a*) No roots (*b*) No roots (*c*) Identify (*d*) Identify

16. (*a*) $2x + 4 = 12$, $x + 1 = 5$, etc. (*b*) $x = 0$, $5x + 7 = 7$, etc.
(*c*) $x + 2 = 1$, $10x + 5 = -5$, etc.

18. (1) No. (2) Yes (3) Yes (4) No

19. (*a*) $4(2x - 1) = 20$, $6y - 3 = 15$, etc
(*b*) $8x - 8 = 8(x - 1)$, $y - 1 = \frac{1}{5}(5y - 5)$, etc
(*c*) $8x + 12 = 8x - 4$, $6x + 12 = 6x$, etc

20. No solution **21.** (*a*) $^{-}12$ (*b*) $^{-}0·6$ (*c*) 3 (*d*) $\frac{1}{2}$

22. 2 **23.** 1 **24.** (*a*) $^{-}2$ (*b*) $^{-}2$ (*c*) 1 (*d*) $^{-}3$

25. (*a*) 1 (*b*) $\frac{6}{11}$ (*c*) 4 (*d*) 2·76 (*e*) ± 8 (*f*) No

26. (1) No solution (2) $\frac{6}{5}$ (3) 5 (4) $\frac{-2}{3}$ (5) 9 (6) $\frac{-87}{57}$

27. 40, 36, 43 **28.** 40, 44, 37

29. Weight of coal 15 ton, water 47 ton and unloaded weight of the engine 60·8 ton

30. 90°, 40°, 50° **31.** 18°, 63°, 99° **32.** base 6 cm and one side 8 cm.

33. 400 ton and 800 ton **34.** 65 ton and 130 ton **35.** 50 litre and 100 litre

36. 8 years **37.** (1) 5·65 (2) 15·5 (3) 2·427 **38.** (*i*) 39, 40, 41
(*ii*) 6, 7, 8, 9 **39.** 62 **40.** 29 **41.** 57 **42.** 1079, 1411

43. 75 km/h, 80 km/h **44.** 40 km/h, 30 km/h **45.** 750 km.

46. 50 ha/day, 700 hactare **47.** 26 km/h, 65 km **48.** 2·1 km, 3·6 km/h

49. 80 km **50.** 1320 km, 230 km/h **51.** 150 Engine **52.** 12 days

53. Copper 100 cubic cm, Iron 50 cubic cm.

54. (1) Rs. 551·25, Rs. 51·25 (2) Rs. 1064·80, Rs 264·80
(3) Rs. 2163·20, Rs 163·20 (4) Rs. 1560·60, Rs. 60·60

56. (1) 2 years (2) 1000 (3) Rs 1040·40 (4) 4%

57. First man will pay 50 paise less

58. Janki, Rs 7 **59.** Rs 1406·08 **60.** Rs. 441 **61.** Rs. 104·25

62. Rs 51 **63.** (1) A (2) *n* (3) P (4) P (5) P (6) *r*

64. Rs. 62 **65.** Rs 5,000 **66.** $\frac{41P}{400} - \frac{P}{10} = Y$, Rs 1600

67. 4% , 2 years , Rs 676 **68.** $P = A \times \frac{40}{41} \times \frac{40}{41} \times \frac{40}{41}$, Rs 2624,000

69. 2 **70.** 10% **71.** Ram paid Rs. 31 more **72.** Khairati , Rs 20

73. Rs 1172 **74.** Rs. 2310 **75.** 4 years

77. {Natural Numbers} **78.** ϕ (empty set)

79. {all rational numbers less than 2}

80. $\left\{\text{all rational numbers less than } \frac{1}{2} \text{ and greater than } \frac{-1}{2}\right\}$

81. (ϕ)

82. $\{x \in I : x > 2\}$

83. $\{x \in Q : x > {}^{-}\frac{1}{2}\}$

84. $\left\{x \in Q : x < \frac{8}{15}\right\}$

85. $\left\{x \in Q : x < \frac{2}{5}\right\}$

86. $\{x \in Q : 6 - x < 5 - x\} = \phi$ **87.** $\{x \in Q : 2x^2 < x^2\} = \phi$

88. $\{x \in I : x > 2\}$

89. $\{x \in I : x < 1\}$

90. $\left\{x \in Q : x > \frac{2}{3} \text{ and } x < 2\right\}$

CHAPTER IV

1. (*a*) 23,000 (*b*) 10·8 (*c*) 25 **2.** (*a*) 640 (*b*) 178

3. (1) 3 (2) $6n$ (3) $9x^2$ (4) $7y^2$

4. (1) $2(x + y)$ (2) $a(x - y)$ (3) $-a(2 + 5b)$

5. (1) $n(m + 1)$ (2) $m(x - 1)$ (3) $5(1 - 3y)$
(4) $5m(n - 1)$ (5) $-5a(3x + 4y)$ (6) $-2n(m + 2)$

6. (1) $a(a - b)$ (2) $x^2(x^2 - 1)$ (3) $y^3(1 - y)$
(4) $p^5(1 - p^2 - p^4)$ (5) $b^3(-b^2 + b + 2)$ (6) $-n^0(1 + n^2 + n^6)$
(7) $6z^4(1 - z^2)$ (8) $xy(x - y)$ (9) $6ax(a + 2x)$
(10) $9b^3(2a - b)$

7. (1) $a^m(1 + a)$ (2) $a^{2n}(a^n - 1)$ (3) $5^{n-2}(5^2 - 1)$

8. (1) $(a - b)(3a + 2b)$ (2) $(p - q)(7q - 2p)$ (3) $(x + y)(2b - c - 2)$

9. (1) $(x - 3)(2m + 5n)$ (2) $m(a - 2)(m - 1)$
(3) $(a - b)(2x + 3y)$ (4) $(x - y)(2a + 1)$

11. (1) $7ax(x - 2a)$, ${}^{-}42$ (2) $(b - c)(a - 2)$, 0

12. (1) 0 ; 4 (2) 0 ; $\frac{{}^{-}2}{7}$ **14.** (1) $(x + y)(2a + 1)$ (2) $(p - q)(k + 1)$
(3) $(a + b)(3b - 1)$ (4) $(x - y)(2y - 1)$

15. (1) $(x - y)(a + b)$ (2) $(x - y)(a + b)$
(3) $(m + n)(m - 5)$ (4) $(x - y)(x - 2)$

16. (1) 2800 (2) 9 **17.** (1) $(x - z)(4x - 3)$
(2) $(3x - 4y)(a + b)$ (3) $(2a - 3y)(5a - 7x)$

18. (1) $(a + b)(ab - 1), 0$ (2) $(y + 3x^2)(x - 2y^2), \frac{11}{36}$

19. $2, {}^{-}2, 3$

21. $(a + b)(x^2 - x + 1)$ (2) $x(x - 1)(a + b + c)$

22. (1) $(x + y)(x - y)$ (2) $(m + n)(m - n)$
(3) $(a + 2)(a - 2)$ (4) $(t + 3)(t - 3)$

23. (1) 7,000 (2) 53,200 (3) 28 (4) 12

24. (1) $(c + 6)(c - 6)$ (2) $(1 + m)(1 - m)$
(3) $(m + 2n)(m - 2n)$ (4) $(6q + 5)(6q - 5)$

25. (1) $(ab + 2)(ab - 2)$ (2) $(7 - pq)(7 + pq)$
(3) $\left(\frac{2}{3}x + \frac{4}{5}y\right)\left(\frac{2}{3}x - \frac{4}{5}y\right)$ (4) $(1 + 0{\cdot}1a)(1 - 0{\cdot}1a)$

26. (1) $(10a^2 + 9b^3)(10a^2 - 9b^3)$ (2) $(a^3 + 2)(a^3 - 2)$

27. (1) $b^2, a^2 - b^2$ (2) $200{\cdot}25$ sq. metre, 1860 sq. m

28. (1) $(a + b + c)(a + b - c)$ (2) $(p - q + r)(p - q - r)$
(3) $(3a + 2b)(3a - 4b)$ (4) $\left(\frac{7}{2}a + 1\right)\left(\frac{5}{2}a + 1\right)$

29. (1) $(p + q - r)(p - q + r)$ (2) $(1{\cdot}2b - c)(0{\cdot}8b - c)$
(3) $(3a + b^2 + c^2)(3a - b^2 - c^2)$ (4) $(1 + 2a - 3b)(1 - 2a + 3b)$
(5) $[2(x - y) + 3]\,[2(x - y) - 3]$ (6) $(4a - 15b)(4a - 25b)$

30. (1) $4mn$ (2) $8xy$ (3) $4m(n + p)$ (4) $4a(b - c)$

31. (1) $\pm\frac{1}{2}$ (2) $\pm 1{\cdot}2$ (3) $\pm\frac{4}{3}$

32. (1) $(x - 1)^2$ (2) $(2a + 1)^2$ (3) $(3m - 1)^2$ (4) $-(a + 1)^2$
(5) $-(a + 3)^2$ (6) $(x^2 - b)^2$ (7) $-(x^2 + n)^2$ (8) $(3c + 2d^2)^2$
(9) $(5x^2 - y^2)^2$ (10) $(6p^2 + q^2)^2$

33. (1) $112ab$; $(8a - 7b)^2$ (2) $120xy$; $(12x + 5y)^2$
(3) $169\,c^8$; $(6a^3b^2 + 13c^4)^2$

34. (1) $(a + b)^3$ (2) $(x - y)^3$ (3) $(m + 2n)^3$
(4) $(a - 2b)^3$ (5) $(4a^2 - b)^3$ (6) $(x^3 - 5y)^3$
(7) $\left(\frac{k}{2} + p^2\right)^3$ (8) $\left(b^5 - \frac{c^2}{3}\right)^3$

35. (1) $150a^2b, 60ab^2, (5a - 2b)^3$ (2) $144x^2y^2$; $108\,xy^4$, $(4x + 3y^2)^3$
(3) $144a^3b^4, 64b^6, (3a^3 - 4b^2)^3$

36. (1) $(2d - p)\,(4d^2 + 2dp + p^2)$ (2) $(4a + b)\,(16a^2 - 4ab + b^2)$
(3) $(4k - 1)\,(16k^2 + 4k + 1)$ (4) $(6x + 1)\,(36x^2 - 6x + 1)$

(5) $(2p + 3q)(4p^2 - 6pq + 9q^2)$ (6) $(5x - 4z)(25x^2 + 20xz + 16z^2)$

(7) $\left(\frac{a}{3} - 10\right)\left(\frac{a^2}{9} + \frac{10a}{3} + 100\right)$

(8) $\left(\frac{2b}{5} + \frac{1}{3}\right)\left(\frac{4b^2}{25} - \frac{2b}{15} + \frac{1}{9}\right)$

(9) $(m - 3)(m^2 + 3m + 9)$ (10) $(1 - q)(1 + q + q^2)$

37. (1) 2,000; 12,200 (2) 5,000

38. (*a*) $\frac{-5}{11}$ (*b*) 10

39. (1) $x(x + 1)(x - 1)$ (2) $5a(a + 1)(a - 1)$ (3) $p^2q^2(p + q)(p - q)$
(4) $5(a + b)^2$ (5) $2a(1 - b)^2$ (6) $(m + n + p)(m + n - p)$
(7) $(x + y + 1)(x + y - 1)$ (8) $(3 + x - y)(3 - x + y)$
(9) $(2 + a + b)(2 - a - b)$ (10) $(a - b)(a + b - 1)$
(11) $(a + b + c + d)(a + b - c - d)$

40. (1) $(x + 1)^2 (x - 1)(x^2 - x + 1)$ (2) $(a + 1)^2 (a^2 - a + 1)$
(3) $(x + 1)(x - 1)(x^2 + x + 1)$ (4) $(x - y)^2 (x + y)$
(5) $(m^2 + n^2)(m + n)(m - n)$ (6) $2b(2a^2 + b^2)$
(7) $(x - 1)^2 (x^2 + x + 1)$ (8) $(c - 1)^2 (c^2 + 1)(c + 1)$

41. (1) $(x - 3)(x - 2)$ (2) $(x - 4)(x + 3)$ (3) $(x + 4)(x - 3)$
(4) $2(a + 2)(a + 3)$ (5) $3(p + 3)(p + 6)$

42. (1) $(x - 3)$ (2) $(a + 2)$ (3) $m - 1$ (4) $-(k + 2)$

43. (1) $4 - mn$ (2) $\frac{1}{2}a - \frac{1}{3}b$ (3) $-\left(\frac{2}{5}x + \frac{3}{4}y\right)$
(4) $-\left(\frac{3}{4}c + \frac{2}{5}d\right)$

44. (1) $x - 2$ (2) $m + n$ (3) $p - q$

45. (1) $4a^2 + 6ab + 9b^2$ (2) $9x^4 - 6x^2y + 4y^2$ (3) $9b^4 - 3b^2 + 1$
(4) $-(4b^4 + 2b^2 + 1)$ Uniform

CHAPTER V

1. (*a*) $\frac{a}{b}$ (*b*) $\frac{5}{x}$ (*c*) $\frac{y}{z}$ (*d*) $\frac{a + b}{4}$ (*e*) $\frac{9}{m - n}$
(*f*) $\frac{x + y}{x - y}$ (*g*) $\frac{3x}{2a + 3b}$ (*h*) $\frac{a^2 - b}{2x^2 + y - 1}$

2. $\frac{1}{a}, \frac{1}{b}; \frac{1}{a} + \frac{1}{b}; \frac{ab}{a + b}$ hours (1) 2·4 hours (2) $\frac{45}{19}$ hours

3. $\frac{a(100 + p)}{100}$; (1) 15750 (2) 72380 (3) 1085000

4. $\frac{vt + v_1t_1}{t + t_1}$, $35\frac{33}{37}$ km/h **5.** $\frac{at}{a - vt}$

6. (1) 0 (2) 0 (3) 0

7. (1) 2 (2) -5 (3) 3 (4) 0, 10 (5) -2, 3 (6) -1, 4

8. (1) 1 (2) -1 (3) c (4) 4 (5) -1

9. 3, 5, impossible, -3, -1, $\frac{1}{3}$, 0, $\frac{3}{5}$, $\frac{3}{4}$

10. (*a*) (1) $\frac{y}{x - 8}$, $\frac{p - q}{3x - 24}$ etc (2) $\frac{10x + y}{14 - 2x}$, $\frac{x}{x - 7}$ etc.

(3) $\frac{x + y}{x - 4}$, $\frac{y}{4 - x}$ and $\frac{x + y}{x + 4}$, $\frac{y}{8 + 2x}$ etc

(4) $\frac{x - y}{x - 5}$; $\frac{3}{10 - 2x}$ etc (5) $\frac{8}{x}$, $\frac{8x + y}{x}$ etc.

(*b*) (1) $\frac{a}{b}$ (2) $\frac{x}{a - 10}$ (3) $\frac{xy^2}{a + 4}$ (4) $\frac{ab}{a - 3}$ (5) $\frac{3x}{a - 20}$

11. (1) 1 (2) -2 (3) 10 (4) 3 **12.** (1)

13. The values of parts (4), (6), (7) will be changed

14. (1) $\frac{2}{3}$, $\frac{3}{8}$, $\frac{2}{5}$, $\frac{29}{81}$ (2) $\frac{3a}{4b}$, $\frac{b}{c}$ (3) $\frac{2n}{3p}$ (4) $\frac{x}{2y}$

(5) $\frac{2a}{3b}$

15. (1) $\frac{1}{y}$ (2) $\frac{3}{7x}$ (3) $\frac{5}{7ab}$ (4) $\frac{2a^2}{3b^4c^7}$ (5) $\frac{15a}{14b^7c}$

16. (1) $\frac{x + y}{3a}$ (2) $\frac{2}{3a^2(x - y)}$ (3) $\frac{a - c}{2}$ (4) 1

17. (1) a^{n-k} (2) b^{3-n} (3) x^{-x} (4) z^{4-k} (5) 4

18. (1) $\frac{2a}{5b}$ (2) $\frac{-8c^2}{15x}$ (3) $\frac{-3x}{-4n}$ (4) $\frac{+x}{+y}$

(5) $-\frac{-3x^3y}{10z}$ (6) $\frac{-4ab}{-7cd}$

19. (1) $-\frac{m}{m - 1}$ (2) $-\frac{b - a}{c + d}$ (3) $-\frac{a - x}{x - b}$ (4) $-\frac{a + b}{c + d}$

20. (1) $-\frac{a}{b}$ (2) $-\frac{1}{3}$ (3) $-\frac{1}{3m}$

21. (1) $\frac{a - b}{a + b}$ (2) $\frac{a + b}{a - b}$ (3) $\frac{a}{a + b}$ (4) $\frac{y}{1 - y}$ (5) $\frac{a - b}{c + d}$

(6) $\frac{k + 1}{x - y}$

22. (1) $\frac{x-2}{x+2}$ (2) $\frac{a^2+ab+b^2}{2}$ (3) $\frac{1-x}{3}$ (4) $-\frac{x}{x+1}$

(5) $-(a+1)$ (6) $\frac{1}{m-n}$

23. (1) $\frac{x+y}{x-y}$ (2) $\frac{a-b}{2(a+b)}$ (3) $\frac{a^2-ab+b^2}{a-b}$

(4) $\frac{m-n}{2(m^2-mn+n^2)}$ (5) $\frac{(x^2+y^2)(x+y)}{x^2+xy+y^2}$ (6) $\frac{1}{a-b}$

24. (1) $\frac{x+y}{x-y}$ (2) $a+b-c$ (3) $\frac{a+b-c}{a+c-b}$ (4) $\frac{(1-y)^2}{z+x}$

25. (1) $\frac{a-b}{a-b-1}$ (2) $\frac{a^2-a+1}{6(a+1)}$ (3) $\frac{3xy}{(3x-y)(x+y)}$ (4) $\frac{ab}{(a+b)^2}$

26. (1) 1 (2) $^{-}1$ or 1 (3) 1 (4) $\frac{1}{m}$

27. (1) $\frac{3}{2}d$ (2) $\frac{b+c}{a}$ (3) $\frac{1+b}{a}$ (4) $2m-3n$

(5) $\frac{a}{b-c}$ (6) $a+1$ (7) $\frac{3(m-n)}{m+n}$

28. (*a*) $\frac{5}{120a^8x^5y}$ and $\frac{2a^2xy}{120a^8x^5y}$ (*b*) $\frac{5xyc^3}{90a^{10}b^6c^3}$ and $\frac{2a^4byz}{90a^{10}b^6c^3}$

(*c*) $\frac{ax^2}{x^{n+2}}$ and $\frac{b}{x^{n+2}}$

29. (*a*) $\frac{m(a+b)}{(a+b)(a-b)^2}$ and $\frac{x(a-b)}{(a+b)(a-b)^2}$

(*b*) $\frac{fb}{ab(bc-ad)}$ and $\frac{ta}{ab(bc-ad)}$

(*c*) $\frac{30}{6(2b-1)(2b+1)}$, $\frac{12(2b+1)}{6(2b-1)(2b+1)}$ and $\frac{9p(2b-1)}{6(2b-1)(2b+1)}$

30. $\frac{5}{7}$, $\frac{a+b}{4}$, a, $\frac{3p^2}{5}$

31. (1) $\frac{2a+b}{3}$ (2) $\frac{2m}{a}$ (3) $\frac{2a}{x+a}$ (4) $\frac{2x+1}{a-2}$

32. (1) $\frac{a-b}{x-1}$ (2) $\frac{m-n}{2p-q}$ (3) $\frac{3}{a-1}$ (4) $\frac{n+4}{p-q}$

33. (1) $\frac{3b+5a}{ab}$ (2) $\frac{53x}{120}$ (3) $\frac{4x}{5}$

34. (1) $\frac{b+2a}{4ab}$ (2) $\frac{34x-21}{15}$ (3) $\frac{26a-3}{12}$

35. (1) $\frac{x(c+b)}{abc}$ (2) $\frac{3(bc^2+2ab^2-a^2c)}{abc}$ (3) $\frac{2m+n}{m^4n^4}$

36. (1) $\frac{b-a^2}{10}$ (2) $\frac{4x+1}{2}$ (3) $\frac{1}{2}$ (4) $\frac{2p}{m}$

37. (1) $\frac{3x}{a-b}$ (2) $\frac{7y^2}{a-2}$ (3) $\frac{3x+2y}{x-y}$ (4) $\frac{a+b}{x^2-1}$

38. (1) $\frac{19}{84}$ (2) $\frac{8x-5y}{40}$ (3) $\frac{-x}{72}$ (4) $\frac{7y-4x}{xy}$

39. (1) $\frac{3b-2}{ab}$ (2) $\frac{xc-yb}{abc}$ (3) $\frac{8p+11q}{30}$ (4) $\frac{13b^2-37a^2}{20}$

40. (1) $\frac{-(x+1)}{4}$ (2) $\frac{x+4}{a-b}$ (3) $\frac{30a^3c-21b^3a+22c^3b}{36a^2b^2c^2}$

(4) $\frac{22a+101b}{24}$ (5) $\frac{2x}{y}$ (6) $\frac{38ab-5ac-36bc}{24abc}$

41. (1) $\frac{mn+1}{n}$ (2) $\frac{x^2+y-xy}{x}$ (3) $\frac{5ab-a-b}{ab}$

(4) $\frac{-b(a+b)}{a}$ (5) $\frac{9x-5y}{8}$ (6) $\frac{x-13y}{4}$

42. (1) 5 (2) $5\frac{1}{4}$ (3) $\frac{1}{5}$ (4) $5\frac{1}{3}$ (5) $2\frac{4}{5}$ (6) $3\frac{1}{3}$

(7) 6 (8) 7

43. (1) 1 (2) $\frac{33}{134}$ (3) 0·8054 (4) $\frac{1}{5}$

(5) $1\frac{32}{35}$ (6) $-\frac{2}{13}$

44. (1) $-\frac{2x+5y}{x(x+y)}$ (2) $-\frac{7x}{10(x-3)}$ (3) $\frac{-1}{12}$ (4) $\frac{2y}{x^2-y^2}$

(5) $\frac{6a}{9a^2-b^4}$ (6) $\frac{2bx}{a^2-b^2}$

45. (1) 0 (2) $\frac{5b^2-a^2}{ab(x+y)}$ (3) $\frac{3mb+2nq}{ab(m+n)}$ (4) $\frac{3(y^2+x^2)}{2x(x^2-y^2)}$

(5) $-\frac{2}{2x+5}$ (6) $-\frac{2}{3z-4}$

46. (1) $\frac{2}{a+2}$ (2) $\frac{-b}{2(a-b)}$ (3) $\frac{c-d}{(c+d)cd}$ (4) $\frac{(x-1)(6x^2+15)}{2x(x+3)(x-3)}$

(5) $\frac{2(n^2-3n-nx-4x)}{(n+2)(n-2)^2}$ (6) $\frac{2(2a^2+5a-13)}{(a+2)(a+3)(a-3)}$

48. (1) $\frac{4a^2+7a-3}{(a+1)^2(a-1)}$ (2) $\frac{-2p^2+12p-45}{2(p+3)(p-3)^2}$ (3) $\frac{4(x+a)}{x+2a}$

(4) $\frac{-12a}{a^4-1}$ (5) $\frac{10(x^2+1)}{(x-1)^2(x+1)^2}$ (6) $\frac{2(a-b)(a^2+b^2)}{(a^2-ab+b^2)(a^2+ab+b^2)}$

(7) $\frac{(x-z)}{x(x+2y)}$ (8) $\frac{11a+c}{6(a-c)}$ (9) $a-1$ (10) 1 (11) abc

(12) $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$

49. (1) $b(a-c)$ (2) $\frac{c}{a}-b$ (3) $\frac{n}{m}-1$ (4) $\frac{c}{a-b}$

(5) $a(a-b)$ (6) $k(2k-n)$ **50.** (1) 1 (2) 80

51. Rs. 901,680. **53.** 3 AM. next day **54.** 10 days (1) $6\frac{2}{3}$ days (2) 5 days

55. (*i*) $\frac{x}{2}$ days (*ii*) $\frac{x}{2}$ days (*iii*) $\frac{x}{3}$ days (*iv*) $\frac{x}{4}$ days.

56. (*a*) 3 hrs. (*b*) 3 hrs. (*c*) 2 hrs. **57.** 24 days **58.** 30 **59.** 25

60. 15 **61.** $\frac{18}{33}$ **62.** 120 km **63.** 240 gm **64.** $\frac{5}{12}$ **65.** $\frac{7}{25}$

66. $\frac{13}{48}$ **67.** (1) $\frac{5}{14}$ (2) $\frac{5x}{14a}$ (3) $\frac{4a}{5c^3}$ (4) $40xa^2$

(5) $\frac{2y^3}{3}$ (6) $\frac{y}{x}$

68. (1) $\frac{-9c}{16a}$ (2) $\frac{-3b}{5}$ (3) $\frac{15xy^2b}{4a}$ (4) $-6a^3b$

(5) $-\frac{x}{4}$ (6) $\frac{2b^5}{5}$

69. (1) $\frac{9b}{5ay}$ (2) $\frac{21m}{31}$ (3) $-2a$ (4) -1

70. (1) $b(a-b)$ (2) $\frac{a^2(a-b)}{a+b}$ (2) $\frac{-3}{4(a+b)}$ (4) $\frac{2}{3}$ (5) $\frac{2}{(x-y)^2}$

(6) $4(a^2+ab+b^2)$ (7) $\frac{x(x+2y)}{3}$ (8) $-\frac{a^2-10ax-b^2+22x^2}{a(2a+b)(a+b-5x)}$

(9) $\frac{p(2p-n)}{n(n-5k)}$ (10) $\frac{x-y}{2x-y-1}$

71. (1) $\frac{b}{3}$ (2) $\frac{3y}{50xz}$ (3) $\frac{4}{9d}$ (4) $-2b^4$

(5) $\frac{-3b^{10}}{a}$ (6) $\frac{3x}{25y^6}$

72. (1) $\frac{5bc}{a}$ (2) $\frac{3a}{cx}$ (3) $\frac{12ayz^2}{25b}$ (4) $\frac{128a^2}{9}$

(5) $\frac{p^2mq}{16a^3b^3c^4x^2y^2}$ (6) $\frac{3b}{4xy}$

73. (1) $\frac{x-y}{2xy}$ (2) $\frac{-a^2(a+2b)}{18b^2(2b^3-x^3a)}$ (3) $\frac{-5x(2x^2-3y^3)}{2y^4(3y^2+2x^2)}$

(4) $\frac{3}{2(1+a^2)}$ (5) $\frac{3(x+y)(x^2-xy+y^2)}{5(x-y)^2}$ (6) $\frac{(a+x)^2(a-x)}{(a^2+ax+x^2)}$

(7) $\frac{15a}{4b(a-b)}$ (8) $\frac{-z(x+3y-2z)}{x(3x+2z)}$

(9) $\frac{5(5b+3)(3a+5b+1)}{(3a-5b)(3a-5)}$ (10) $\frac{z(x-4z)}{x(x-5)}$

74. (1) $\frac{1}{5}$ (2) $\frac{3}{10}$ (3) $\frac{a-b}{a+b}$ (4) $\frac{2x-y-5a}{2x+y+5a}$ (5) $\frac{y+x}{y-x}$

(6) $\frac{a^2+b^2}{a^2-b^2}$ (7) $-\frac{x}{a}$ (8) $\frac{1}{x}$

75. (1) 1 (2) 1 (3) 1 (4) 1

76. (1) $\frac{1}{16}$ (2) $\frac{16}{81}$ (3) $\frac{x^5}{y^5}$ (4) $\frac{25a^2}{16b^2}$

(5) $\frac{a^{10}b^2}{c^2d^4}$ (6) $\frac{125c^6d^9}{27a^9b^9}$

77. (1) $\frac{-n^{18}k^9}{1000p^{12}}$ (2) $\frac{a^{4n}}{b^{6n}}$ (3) $(-1)^n\frac{a^n}{x^{3n}}$ (4) $(-1)^{2k-1}\frac{b^{4k-2}}{c^{2k-1}}$

(5) $\frac{-a^{12}}{3x^4y^6}$ (6) $\frac{-c}{8b}$

78. (1) $13\frac{1}{3}$ (2) 2 (3) -2 (4) $3-x^3$ (5) $\frac{(x+y)^2}{x+y+1}$

79. (1) -1 (2) $\frac{xa}{(x-a)(x+a)}$ (3) $-\frac{a+b}{a-b}$

(4) $\frac{(m+n)^2}{m^2-n^2-m^2n}$ (5) -1

80. (1) $\frac{a^2+ab+b^2}{a-b}$ (2) $\frac{-2a}{4a^2+b^2+1}$

(3) $m-m^2+1$ (4) $\frac{5p-2}{6(2-p)(p-1)}$

82. (1) 5 (2) $\frac{-3}{5}$ (3) $3\frac{4}{7}$ **83.** (1) yes (2) yes.

84. (1) 1 (2) 4 (3) $\frac{1}{2}$ (4) 0 (5) 1 (6) $\frac{-2}{3}$

85. $\frac{-17y+5}{2y+17}$, $\frac{-17x+5}{2x+17}$

86. (1) $\frac{an-cm}{mn}$ (2) 0 (3) $\frac{-a^2}{h}$

(4) $\frac{c^2-t^2}{7c+5t}$ (5) $\frac{2-a^2}{a}$

87. $2c - b$ **88.** (1) $\frac{b - c}{b + c}$ (2) $\frac{c(1 \text{ [illegible]}}{1 - a}$ (3) $\frac{b(1 - a)}{1 + a}$

90. $\frac{3}{\text{[illegible]}}$ **91.** 28, 40

92. 12 km/h, 30 km/h, **93.** 6 hrs. **94.** 80 km/h

95. 400 km/h, 1200 km/h. **96.** 25 and 40, 100 metre.

97. 144 revolutions **98.** 7 labourers **99.** 40 days, 60 days

100. 20 tools, 28 tools **102.** 70 ha, 50 ha

103. At $\frac{3}{2}$ metres from the heavy boy

104. 760 gm gold, 300 gm silver. **105.** 39200 cubic cm.

106. $3\frac{9}{13}$ days, $7\frac{5}{13}$ days, $1\frac{11}{13}$ days

107. $2\frac{2}{7}$ hrs, $6\frac{6}{7}$ hrs. **108.** 3 hrs **109.** 16 days.

CHAPTER VI

4. (1) I quadrant (2) II quadrant (3) III quadrant (4) IV quadrant
(5) to the left of x—axis (6) on y—axis above the origin (6) origin
(8) on y —axis below the origin

5. (1) I quadrant (2) III quadrant (3) IV quadrant (4) II quadrant
(5) on y—axis below the origin (6) on x—axis to the left (7) origin

7. $4, {}^-6$ **8.** 2, 3 **10.** (1) $(1, {}^-3), ({}^-1, 3)$ (2) $(2, {}^-5), ({}^-2, 5)$
(3) $(1, 3), ({}^-1, {}^-3)$ (4) $(2, {}^-5), ({}^-2, 5)$ (5) $({}^-1, {}^-3), (1, 3)$.

15. $y = 4x$, **16.** $x = 2y$ **17.** (1) $y = 4x$.

ERRATA

Page	*Problem*	*For*	*Read*
27	23	$1 < 2^2 < 10$	$1 < x^2 < 10$
39	75	$-8m - 3m$	$-8m + -3m$
46	114(2)	$(-15y^5)$	$(-15y^8)$
97	23(*d*)	$-2{\cdot}35z - 5.381\ldots$	$-2{\cdot}35z = 5{\cdot}381\ldots$
128	39(*ii*)	$5a^3$	$5a^2$